Instituut voor Theoretische Fysica, Universiteit Utrecht

FINAL EXAM QUANTUM FIELD THEORY

January 31, 2013

- The duration of the exam is 3 hours.
- The exam is closed-book.
- Usage of a calculator and a dictionary is allowed.
- Use different sheets for each exercise.
- Write your name and initials on every sheet handed in.
- Divide your available time wisely over the exercises.

Problem 1 (20 points)

Starting from the *Eulcidean path integral* for the vacuum-vacuum transition amplitude for the harmonic oscillator in the presence of a time-dependent external force $F(t)$

$$Z_E[F] = \int \mathcal{D}q\, e^{-\int \mathrm{d}\tau \left[\frac{1}{2}(\frac{dq}{d\tau})^2 + \frac{\omega^2}{2}q^2 - Fq\right]},$$

1. show that

$$Z[F] = Z_E[0] \exp\left[\frac{1}{2}\int \mathrm{d}\tau \mathrm{d}\tau' F(\tau) D_E(\tau - \tau') F(\tau')\right];$$

2. work out an expression for $D_E(t)$.

Problem 2 (10 points)

Let $\psi(x)$ be a free Dirac field. Use the Wick theorem to evaluate the following correlation function

$$\langle 0|T(:\bar{\psi}_i(x)\psi_j(x) :: \bar{\psi}_k(y)\psi_l(z):)|0\rangle\,.$$

Problem 3 (15 points)

Show that the massless Dirac Lagrangian

$$\mathscr{L} = \frac{i}{2}\left(\bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma^\mu\psi\right)$$

is invariant with respect to the so-called chiral transformations

$$\psi \to e^{i\alpha\gamma^5}\psi\,, \qquad \alpha \in \mathbb{R}\,.$$

Find the corresponding Noether current and verify that it conserves due to the Dirac equation.

Problem 4 (30 points)

Consider the scalar ϕ^3 theory in *four-dimensional* space-time governed by the Lagrangian

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{g}{3!}\phi^3\,.$$

1. Use dimensional regularization to regularize the following graph contributing to the self-energy;

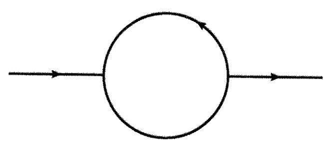

Figure 1: Divergent diagram at order g^2 in the ϕ^3 theory.

Hint. Introduce a mass parameter μ in such a way that in d-dimensions the coupling g would retain its canonical dimension which it has in four dimensions.

2. Compute the corresponding integral by using the Feynman formula. Remember also that

$$\int \frac{d^dp}{(p^2+2pQ-M^2)^\alpha} = (-1)^\alpha\, i\pi^{\frac{d}{2}}\,\frac{\Gamma\left(\alpha-\frac{d}{2}\right)}{\Gamma(\alpha)}\,\frac{1}{(Q^2+M^2)^{\alpha-\frac{d}{2}}}\,.$$

3. Find the corresponding renormalization of the bare mass.

Problem 5 (15 points)

In ϕ^4 theory, the renormalization at one loop of the bare mass m_0 in the minimal subtraction scheme is found to be

$$m_0^2 = m^2 \left(1 + \frac{\frac{1}{16\pi^2} g}{\epsilon}\right),$$

while the β-function is

$$\beta(g) = -\epsilon g + \frac{3g^2}{16\pi^2} + \dots$$

Here g and m are the renormalized mass and the coupling constant, and ϵ is a regularization parameter of dimensional regularization. Using the formula for m_0 together with the expression for the β-function, compute the one-loop anomalous dimension

$$\gamma_m(g) = \mu \frac{\partial \log m}{\partial \mu}.$$

Problem 6 (10 points)

Consider the Lagrangian density for a massive vector field (work in units $\hbar = c = 1$)

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu .$$

Find the Hamiltonian of the model and the corresponding Hamiltonian equations of motion.