Instituut voor Theoretische Fysica, Universiteit Utrecht

MID-TERM EXAM QUANTUM FIELD THEORY

November 7, 2013

- The duration of the exam is 3 hours.
- The exam is closed-book.
- Usage of a calculator and a dictionary is allowed.
- Use different sheets for each exercise.
- Write your name and initials on every sheet handed in.
- Divide your available time wisely over the exercises.

Problem 1 (25 points)

The interaction between a Dirac fermion of mass m and a real scalar field of mass k is governed by the theory with the action

$$S = \int d^4x \Big(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{k^2}{2}\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \lambda\phi\bar{\psi}\psi \Big) ,$$

where λ is the coupling constant.

1. Derive the classical equations of motion.
2. Compute the stress-energy tensor.

   Hint. Use the general formula that follows from the Noether theorem

   $$T^k_n = \frac{\partial \mathscr{L}}{\partial(\partial_k\phi_I)}\partial_n\phi_I - \delta^k_n\mathscr{L} .$$

   Here ϕ_I is a set of all fields entering a given action with the Lagrangian density $\mathscr{L}$.
3. Restore the physical dimensions of all the quantities entering the action S and determine the physical dimension of λ.

Problem 2 (20 points)

Consider a real scalar field with the Lagrangian density

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2 \, .$$

Then write

$$\phi(x) = \int \mathrm{d}\vec{k} \left[a(\vec{k}) e^{-ik\cdot x} + a(\vec{k})^\dagger e^{ik\cdot x} \right] \, ,$$

and recall that the stress-energy tensor is defined as in **Problem 1.**

1. Express the momentum operator $P^i = \int \mathrm{d}\vec{x}\, T^{0i}$ in terms of the modes $a(\vec{k}), a(\vec{k})^\dagger$.
2. Assume that in the quantum theory the corresponding operator P is normal ordered and determine the commutation relations of the operators $a(\vec{k}), a(\vec{k})^\dagger$ from the requirement that

$$[P^i, \phi] = -i \frac{\partial \phi}{\partial x_i} \, .$$

Problem 3 (10 points)

Consider a complex scalar field with the Lagrangian

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi^* - \frac{m^2}{2}\phi\phi^* \, .$$

Express the Feynman propagator $\langle \phi^*(x)\phi(y) \rangle$ in terms of the standard expression

$$\triangle(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{-ikx} \, .$$

Problem 4 (25 points)

Determine those elements of the rotation group which commute with the helicity operator

$$\Sigma = \frac{1}{|\vec{p}|} \gamma^0 \gamma^5 \gamma^i p_i \, ,$$

where $\vec{p} \equiv \{p_i\}$, $i = 1, 2, 3$ is the momentum of a Dirac fermion.

Problem 5 (20 points)

1. Show that the charge conjugated spinor $\psi^c(x)$ transforms under proper orthochronous Lorentz transformation in the same way as $\psi(x)$, that is

$$\psi^{c'}(x') = S\,\psi^c(x)\,,$$

where S is a matrix of Lorentz transformations.

2. Determine the transformation law of the charge conjugated spinor $\psi^c(x)$ under parity.