Quantum Field Theory 2008/09 – Final Exam, 29 Jan 2009, 14:00-16:00h

You must obtain at least 10 of the 20 points to pass the exam. The use of auxiliary materials such as books, notes, calculators, laptops etc. is not permitted. Please hand in all sheets you used for calculations.

Problem 1 - Photon propagator (10 points)

Consider the correction to lowest order in the coupling constant to the photon propagator in QED, given by the amplitude of the diagram shown in Fig. 1.

(a) Write down the truncated amplitude (i.e. omitting the propagators of the two external photon legs) associated with this Feynman diagram.

(b) Regularizing any infinities by introducing a Lorentz-invariant momentum cut-off Λ, $\Lambda \gg m$, determine the leading, nonvanishing term in Λ contributing to the amplitude by performing all integrations explicitly. (Allow for finite variable shifts in the momentum integrals.)

(c) Does the superficial degree of divergence of the diagram coincide with that found in your calculation?

[Hints: The identity

$$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{(xA + (1-x)B)^2}, \tag{1}$$

may come in handy. – If you have trouble recalling γ-matrix identities, appeal to their tensorial structure. For example, we must have tr $\gamma^\kappa \gamma^\lambda \gamma^\mu \gamma^\nu = \alpha_1 \eta^{\kappa\lambda}\eta^{\mu\nu} + \alpha_2 \eta^{\kappa\mu}\eta^{\lambda\nu} + \alpha_3 \eta^{\kappa\nu}\eta^{\lambda\mu}$ for some constants α_i.]

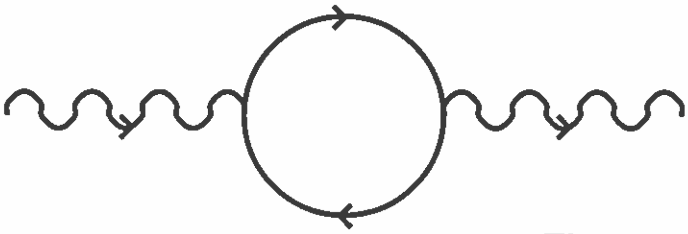

Fig. 1

Problem 2 - Vacuum energy (6 points)

Compute the vacuum energy of a free massive quantum scalar field in four space-time dimensions in a canonical operator picture, starting from the classical Lagrangian.

Problem 3 - Mimicking four-fermion interactions (4 points)

The Lagrangian

$$\mathcal{L}(W_\mu, \Psi) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 W_\mu W^\mu + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + gW_\mu(\bar{\psi}\gamma^\mu\psi), \qquad (2)$$

where $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, describes a simplified version of the standard model, and captures the manner in which the standard model gives rise to an effective four-fermion interaction.

(a) Give a complete set of Feynman rules in momentum space for this theory.

(b) Consider only tree diagrams which satisfy (i) all external lines are fermions, (ii) $p^2 \ll M^2$ for all momenta p_μ. Show that such diagrams can effectively be described by

$$\mathcal{L}_{eff}(\psi) = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \frac{\lambda}{(2!)^2}(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi), \qquad (3)$$

and relate λ to the parameters of the Lagrangian (2).