Departement Natuur- en Sterrenkunde
Universiteit Utrecht

RETAKE EXAM Quantum Field Theory - NS-TP401M

Thursday, April 20, 2017, 13:30-16:30, BBG079.

1) Start every exercise on a separate sheet. Write on each sheet: your name and initials, and your studentnumber.
2) Please write legibly and clear. Unreadable handwriting cannot be marked!
3) The exam consists of **three** exercises and replaces the total final mark.
4) No lectures notes or any other material (books, calculators, ...) are allowed. A formularium instead is given.

Formularium

In this exam we use natural units, in which $c = \hbar = 1$. The Minkowski metric in four spacetime dimensions is $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. The Dirac matrices $(\gamma^\mu)_\alpha{}^\beta$ satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\,\eta^{\mu\nu}\,1_{4\times 4}\ . \tag{1}$$

The Delta function has an integral representation given by

$$\int_{-\infty}^{+\infty} \mathrm{d}p\, e^{ipx} = 2\pi\delta(x)\ . \tag{2}$$

In quantum mechanics, canonical commutation relations between coordinates q and momenta p are given by $[q, p] = i$.

1. Short questions (3 points)

Answer the following questions (be brief when you can):

i) What is a chiral spinor (or Weyl spinor, or left- and right-handed spinors), what is chiral symmetry and what does it have to do with massless fermions?

ii) Why do we need anticommutation relations for fermions instead of commutation relations? What goes wrong if we use commutation relations for fermions?

iii) What is Feynman's $i\epsilon$-prescription for propagators in quantum field theory, and why is it needed? What does it have to do with the Wick rotation?

iv) How do fermions interact with the electromagnetic field? Answer this by writing down interaction terms in the Lagrangian. Repeat this for scalar fields, how do they interact with the electromagnetic field?

2. Two scalar fields (3 points)

Consider the Lagrangian density of two real scalar fields ϕ_1 and ϕ_2,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2) - \frac{1}{2}m^2(\phi_1^2 + \phi_2^2) - \lambda(\phi_1^2 + \phi_2^2)^2 \ . \tag{3}$$

i) Write down the equations of motion for the two scalar fields.

ii) Compute the momenta π_1 and π_2 conjugate to ϕ_1 and ϕ_2 and determine the Hamiltonian H.

iii) Quantize the theory by imposing equal time commutation relations between fields and momenta. Determine the equations of motion in the Hamiltonian picture by computing (for $a = 1, 2$)

$$\dot{\pi}_a = -i\,[\pi_a, H] \ , \qquad \dot{\phi}_a = -i\,[\phi_a, H] \ , \tag{4}$$

and show that this reproduces the equations of motion of the classical theory in the classical limit, obtained in item i).

3. Renormalizability of Yukawa couplings? (4 points)

Consider a field theory for a massive Dirac spinor ψ and a scalar field ϕ in $d = 3$ spacetime dimensions. The free Lagrangian (density) reads

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 \ . \tag{5}$$

We consider two types of interactions between these fields,

$$\mathcal{L}_1 = g_1\,(\bar{\psi}\psi)\,\phi \ , \qquad \mathcal{L}_2 = g_2\,(\bar{\psi}\gamma^\mu\psi)\,\partial_\mu\phi \ . \tag{6}$$

i) Write down the Feynman rules (in momentum space) for the two theories given by $\mathcal{L}_0 + \mathcal{L}_1$ and $\mathcal{L}_0 + \mathcal{L}_2$.

ii) Consider the one-loop diagrams contributing to the fermion self-energy and give the explicit expressions for the two theories. Are the diagrams divergent and, if so, indicate the counterterms that one needs to absorb all the (one-loop) divergencies.

iii) What are the mass-dimensions of the fields and the coupling constants in the two theories? Are these theories renormalizable by power counting and why (not)? If not, write down a Feynman diagram that leads to a counterterm that cannot be absorbed in the original Lagrangian.