Mid-term Cosmology, 22 May 2019.

Constants of nature are given by $h = 6.6\,10^{-34} m^2 kg/s$, $G_N = 6.7\,10^{-11} Nm^2/kg^2$, $c = 3\,10^8 m/s$ and $k_B = 1.38\,10^{-23} m^2 kg\, s^{-2} K^{-1}$.

1 Life in different times

In a short paper[1] Avi Loeb argued against the anthropic principle by posing that intelligent life may exist early on in our universe when the CMB was a comfortable $0-100$ degrees Celsius ($273-373$ degrees Kelvin).

a) [9] Assuming a matter-dominated universe, how long did this comfortable period last? Use that the universe is approximately 13.7 Gyr old and that the current CMB temperature is 2.72 K.

b) [7] The paper above also argues that there locally may be enough heavy elements from early supernovas to make habitable planets with water. If this really allows for intelligent life, why would this then give an argument against Weinberg's anthropic explanation of the cosmological constant problem?

c) [8] In a similar argument he also argued that there may be cosmologists around in the far future, in a trillion (10^{12}) years from now. Assume the universe is dominated by a cosmological constant with the current Hubble constant (about 70 km/s/Mpc); what would be the (order of magnitude) wavelength of a CMB photon at that time? What do you think experimental cosmology would be like at such a time? You may use that current CMB photons have $\lambda \sim 1$mm.

d) [4] Approximately when did recombination, last scattering, nucleosynthesis and inflation take place? Spend about half a sentence on each to describe what happened.

– 2 Exotic dark matter and a universe without singularities

In this problem we explore a closed universe with curvature radius R_0 with **negative** cosmological constant Λ and an exotic form of dark matter. This dark matter has an equation of state $p = -\frac{2}{3}\epsilon$, with p the pressure and ϵ the energy density.

a) [5] Show that the relation between ϵ and the scale factor $a(t)$ for this dark matter is given by $\epsilon(t) = \epsilon_0/a(t)$.

b) [10] This universe has no big bang singularity, nor a singularity in the future. Instead, show by using the acceleration equation that a solution to the Friedmann equation is given by an oscillating universe, $a(t) = a_0 + A\cos(\omega t)$. What are ω and a_0 in terms of Λ and ϵ_0?

c) [8] At what Λ does this universe become static? Would such a universe be stable?

d) [8] Now use the other Friedmann equation to obtain A. What happens with the ratio between the maximum and minimum scale factors during the oscillations in the limit $|\Lambda| \to 0$?

3 A cosmic discrepancy

As stated in the lectures a few times there is a discrepancy between direct measurements of the current Hubble consant (73.2 ± 1.7km/s/Mpc, by Gaia) and by the value measured by the Planck satellite (67.4 ± 0.5 km/s/Mpc) that recently became significant at 3.8 standard deviations.

a) [6] Figure 1 shows old data of luminosity distance and redshifts of type 1a supernovas. Explain in words how both of these are measured, whereby you also include the two reasons why the luminosity distance $d_L = d_p(1+z)$, with d_p the physical distance.

[1] Abraham Loeb, The Habitable Epoch of the Early Universe, 1312.0613

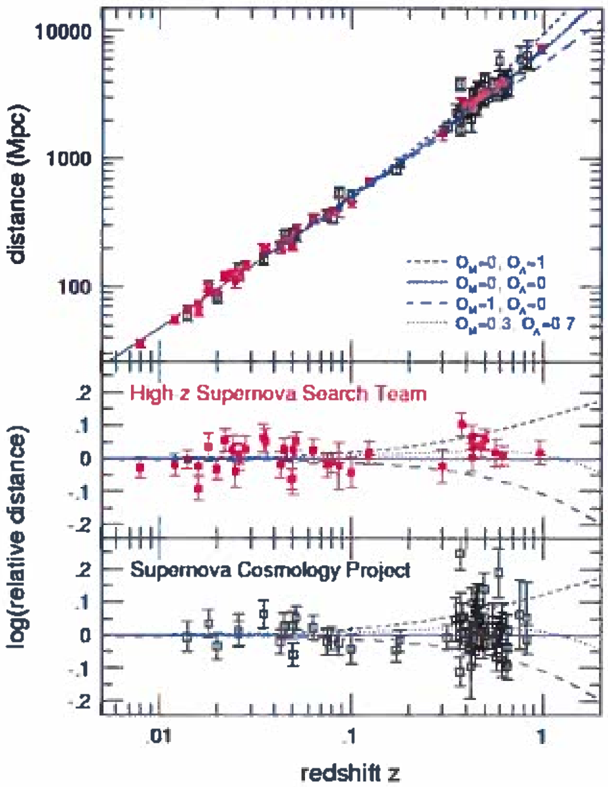

Figure 1: Diagram displaying distance vs. redshift for Type Ia Supernovae. The points are the supernova measurements assembled by two independent groups, the High-z Supernova Search Team (Riess et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1999).

b) [10] Assume a constant expansion of a flat universe ($\dot{a}$ constant). Use the Doppler-formula ($f_{obs} = f_{emi}\sqrt{\frac{1-\beta}{1+\beta}}$, with f_{obs} and f_{emi} the observed and emitted frequencies, and $\beta = v/c$ the fraction of the speed of light at which the object is receding) to derive a formula for the luminosity distance d_L in terms of the redshift z and the Hubble constant H_0. Is the approximation $\beta \approx z$ still reasonable at $z = 1$?

c) [7] From the figure extract the value of the Hubble constant at $z = 0.01$, 0.1 and 1.0. Why is it clear from the figure that the universe is currently accelerating? [if you were not able to do (b) use may use the standard Hubble law, assuming $\beta \approx z$]

d) [7] Explain in words how the Planck collaboration obtains its value of the Hubble constant. Use about three to five sentences.