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In 2006/2007, the course NS-TP430m was given by dr. T. Prokopec.

Cosmology (NS-TP430m)
June 28, 2007

Question 1: Theoretical questions *(4 points)*

a) Transitions in the Early Universe *(1 point)*

Explain the difference between a first order transition, a second order transition and a crossover. What is known about the nature of (a) the electroweak transition and (b) QCD transition?

b) Causality problem *(1 point)*

Formulate the causality problem in cosmology. *Hint:* Make use of the CMB anisotropies.

c) Domain walls *(1 point)*

Consider an early Universe phase transition at which domain walls form. Explain why is the late time scaling behavior of domain walls $\rho_{\rm dw} \propto 1/a$, and why could formation of domain walls be disastrous for our existence.

d) Cosmological perturbations *(1 point)*

Explain the physical process by which inflation generates scalar cosmological perturbations.

Question 2: Redshift *(4 points)*

What is the redshift at the QCD transition? Recall that QCD transition occurred at a temperature $T_{\rm QCD}$ 160 MeV$/k_B$. Assume that the three lightest quarks (up, down and strange) and the lightest charged lepton are relativistic and that the other three quarks ($m_b, m_c, m_t \gg T_{\rm QCD}$) and the other charged leptons are very heavy. Do we have any direct evidence from the QCD transition?

Question 3: Chaotic inflation *(4 points)*

Consider the slow roll regime of the chaotic inflationary model with the scalar field potential,

$$V = \frac{\lambda_6}{6!}\,\frac{\varphi^6}{M_P^2}. \tag{1}$$

a) Starting with the equation of motion for the inflaton and the Friedmann equation written in slow roll regime, calculate $\varphi = \varphi(t)$. *(1 point)*

b) Calculate the scale factor $a = a(t)$. *(1 point)*

c) Calculate the number of e-foldings N and express it in terms of (one of) the slow roll parameters ϵ or η defined as

$$\epsilon = \tfrac{1}{2}M_P^2\left(\frac{{\rm d}V/{\rm d}\phi}{V}\right)^2, \qquad \eta = M_P^2\,\frac{{\rm d}^2V/{\rm d}\phi^2}{V}, \tag{2}$$

where $V = V(\phi)$ denotes the inflaton potential, and $\phi = \phi(t)$ is the inflaton field. *(2 points)*

Question 4: Cosmological perturbations *(4 points)*

The amplitude of scalar and tensor cosmological perturbations in slow roll approximation is conveniently expressed in terms of the corresponding spectra,

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{1}{\epsilon}\frac{H^2}{8\pi^2 M_P^2}\right)_{1X}$$
$$\mathcal{P}_g = \left(\frac{H^2}{8\pi^2 M_P^2}\right)_{1X}, \tag{3}$$

where H denotes the Hubble parameter during inflation, $M_P = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, the subscript $1X$ indicates that the quantity needs to be evaluated at the first Hubble crossing during inflation and $\epsilon \equiv -\dot{H}/H^2 (\dot{H} = \mathrm{d}H/\mathrm{d}t)$ is a slow roll parameter. Because of the time dependence of the Hubble parameter during inflation, the amplitude of the spectra (3) exhibit a dependence on the physical momentum of perturbations, which to leading approximation has a power-law form,

$$\mathcal{P}_{\mathcal{R}} \propto k^{n_s-1}, \quad \mathcal{P}_g \propto k^{n_g}. \tag{4}$$

where $n_s - 1$ and n_g denote the spectral indices of the scalar and tensor cosmological perturbations. Show that to leading order in slow-roll approximation spectral indices can be expressed in terms of slow roll parameters as follows,

$$n_g = -2\epsilon, \qquad n_s - 1 = -6\epsilon + 2\eta, \tag{5}$$

where ϵ and η are defined in Eq. (2).
Hint: Make use of $d\ln(k) = d\ln(aH) = H(1-\epsilon)dt = H(1-\epsilon)d\phi/\dot{\phi}$.

Question 5: The growth of density perturbations *(4 points)*

The density contrast δ of a fluid with an energy density ρ is defined by

$$\rho(\vec{x},t) = \rho_0(t)\left(1 + \delta(\vec{x},t)\right). \tag{6}$$

In an expanding spatially flat universe, in which evolution of the scale factor $a = a(t)$ is given by the Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho_0(t), \tag{7}$$

the density contrast obeys the equation

$$\ddot{\delta} + 2H\dot{\delta} + \left[-c_s^2\frac{\nabla^2}{a^2} - 4\pi G_N \rho_0(t)(1+w)(1+3w)\right]\delta = 0, \tag{8}$$

where $\nabla^2 = \sum_{i=1}^{3}\left(\frac{\partial}{\partial x_i}\right)^2$, $w = \mathcal{P}_0/\rho_0$ is defined by the equation of state, $c_s^2 = \frac{\partial \mathcal{P}_0}{\partial \rho_0}$ defines the speed of sound of the fluid and $\dot{\delta} = \frac{\mathrm{d}\delta}{\mathrm{d}t}$.

a) Show that in a space-time where $w =$ constant the two solutions of Eq. (8) for the density contrast in (spatial) Fourier space $\tilde{\delta} = \tilde{\delta}(\vec{k},t)$ are of the form,

$$\tilde{\delta} = At^{\frac{2}{3}\frac{1+3w}{1+w}} + Bt^{-1}, \qquad (k \ll k_J) \tag{9}$$

where A and B are (time independent) constants and

$$\frac{k_J}{a} = \left(\frac{4\pi G_N \rho_0(t)}{c_s^2}\right)^{\frac{1}{2}} \tag{10}$$

denotes the physical Jeans momentum. The first solution is the growing mode while the second is the decaying mode. *(2 points)*

b) What is the growth rate of the density contrast δ (a) in radiation era and (b) in matter era? Estimate the growth factor for δ from the radiation-matter equality to today. *(1 point)*

c) Has the growth of structure sped up or slowed down during the era of recent cosmic acceleration? Could that effect be used for a measurement of the cosmic acceleration today? *(1 point)*