INSTITUUT VOOR THEORETISCHE FYSICA
UNIVERSITEIT UTRECHT

MIDTERM EXAM String Theory

Tuesday, April 19, 2011.

1) Start every exercise on a separate sheet.

2) Write on each sheet: your name and initials, and your student number.

3) Please write legibly and clear! Unreadable handwriting cannot be marked!

4) The exam consists of **two** exercises. You are not allowed to use lecture notes or any other material.

1. Mixed Dirichlet-Neumann boundary conditions

Consider the open string, with worldsheet coordinates $\sigma^\alpha = (\tau, \sigma); \alpha = 1, 2$ and $\sigma \in [0, l_s]$, where l_s denotes the string length. The field equation for a single string coordinate X reads

$$\partial_\alpha \partial^\alpha X = 0 \tag{1}$$

- Write down the oscillator mode expansion for X satisfying the field equation, with mixed boundary conditions (DN)

$$X(\tau, \sigma = 0) = a \ , \qquad \partial_\sigma X(\tau, \sigma = l_s) = 0 \ , \tag{2}$$

  where a is some arbitrary real constant. Hence, the string has Dirichlet boundary conditions on one end, and Neumann boundary conditions on the other end.

- From the reality of X, derive a reality condition on the oscillator modes.

- What is the mode expansion for the case of mixed boundary conditions (ND)

$$\partial_\sigma X(\tau, \sigma = 0) = 0 \ , \qquad X(\tau, \sigma = l_s) = a \ . \tag{3}$$

  [Note: if you can guess the answer without calculation, that's fine. Motivate your guess briefly.]

2. Closed string states

Consider the closed string in light-cone gauge, in which the only independent transversal oscillator modes are denoted by α_n^i and $\tilde{\alpha}_n^i$, where $i = 1, ..., D-2, (D = 26)$, and with non-vanishing commutation relations $[\alpha_m^i, \alpha_n^j] = [\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] = m\delta_{m+n,0}\delta^{ij}$. The Fock vacuum $|0\rangle >$ satisfies $\alpha_n^i|0\rangle = \tilde{\alpha}_n^i|0\rangle = 0; n > 0$, and the mass operator is given by (in dimensionless units)

$$M^2 = 8(\sum_{m=1}^{\infty} \alpha_{-m}^i \alpha_m^i - 1) = 8(\sum_{m=1}^{\infty} \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i - 1) \ , \tag{4}$$

where in the last equation we have used the Virasoro constraints, and a sum over the transversal modes is understood.

2a. States in D=26

- Consider the states
$$\alpha_{-p}^i \alpha_{-q}^j \tilde{\alpha}_{-r}^k |0\rangle \ , \tag{5}$$
where the mode numbers p, q, r are all positive (and nonzero) integers. What are the possible values of the mode numbers p, q, r for which (5) is a physical state ? Compute the mass $M_{(p,q,r)}$ of these states.

- Fix $r = 3$. How many states do you count as a function of $d = D - 2$? What is your answer for $r = 4$?

- Reinstall the string tension T (which has the dimension of a Newton), Planck's constant $\hbar$ (with dimension m^2kg/s), and the speed of light c in the mass formula such that M has the dimension of gram.

2b. Compactification on a torus

The mass formula (4) holds in $D = 26$. Upon compactifying on a circle of radius R, the mass formula in the remaining 25 dimensions becomes

$$M_{25}^2 = \left(\frac{n}{R} + 2mR\right)^2 + 8(\sum_{m=1}^{\infty} \alpha_{-m}^i \alpha_m^i - 1) = \left(\frac{n}{R} - 2mR\right)^2 + 8(\sum_{m=1}^{\infty} \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i - 1) \ , \tag{6}$$

where n is the momentum mode, and m is the winding mode.

- Based on (6), write down the mass formula in 24 dimensions after compactifying further on a second circle of radius $\tilde{R}$, i.e. what is the mass formula after compactification on a torus? Explain your answer.

- Consider again the states (5) with mass $M_{(p,q,r)}$ for fixed p, q, r. How many scalars are produced from these states with the same mass, now in the 24-dimensional spacetime ? How many vectors ? Write down the corresponding states.