INSTITUUT VOOR THEORETISCHE FYSICA
UNIVERSITEIT UTRECHT

FINAL EXAM String Theory

Friday, July 5, 2013.

1) Start every exercise on a separate sheet.
2) Write on each sheet: your name and initials, and your student number.
3) Please write legibly and clear.
4) The exam consists of **four** exercises. You are allowed to use lecture notes, books or any other handwritten or printed document but not computers or calculators, nor the internet.
5) Show at least some details of your calculations; sheets with answers just copied from the notes will not be graded.

1. Fermionic current

a. By using the equations of motion of the fermionic string in the super-conformal gauge, show the conservation of the fermionic current

$$G_\alpha = \frac{1}{4}\rho^\beta \rho_\alpha \psi^\mu \partial_\beta X_\mu .$$

b. Show that the conservation equations in the light-cone variables lead to infinitely many of conserved charges.

c. Expand the components of G_α in the usual Virasoro basis by defining

$$G_r = \frac{1}{\pi}\int_0^{2\pi} e^{-ir\sigma} G_-, \qquad \tilde{G}_r = \frac{1}{\pi}\int_0^{2\pi} e^{-ir\sigma} G_+$$

and express G_r and $\tilde{G}_r$ in terms of the oscillator modes of X^μ and ψ^μ.

d. Work out the commutation relation $[L_m, G_r]$ where L_m are the (normal-ordered) Virasoro generators of the fermionic string.

2. Massive string spectrum

a. Write down all the string states of the *open* fermionic string in the *first two (positive) massive* levels in the NS sector and *the first massive* level in the R sector. Calculate

their masses.
b. What are the total fermonic and bosonic degrees of freedom (in space-time) at a given mass level? Are they equal in number?
c. Which of these states are left in the spectrum after the GSO projection? Are the fermionic and bosonic degrees of freedom equal in number after the GSO projection?

3. Orbifolds

Consider the closed bosonic string. In Problem Set 13.2 you worked out the circular compactification of the bosonic string theory where one of the space coordinates, X is compactified on a circle with radius R as $X \equiv X + 2\pi R$.

Now, in addition to this compactification on the circle S^1, assume that there is also the "reflection" symmetry $X \equiv -X$.
a. What are the fixed points (points on X that are mapped onto themselves) under *both* the compactification *and* the reflection above.
b. In problem 13.2 you have shown that there are new sectors in the string spectrum when there is a compact dimension, namely the *winding modes* $X(\sigma + 2\pi) = X(\sigma) + 2\pi R w$ with $w \in \mathbf{Z}$. By the same token, there will be new sectors in this background, not only the "winding" sectors but also the "twisted" sectors: $X(\sigma + 2\pi) = -X(\sigma)$.
i. Derive the oscillator expansion of $X(\sigma, \tau)$ in the twisted sector.
ii. What is the center of mass coordinate, what is the center of mass momentum?
iii. How many twisted sectors are there?

4. String scattering

Compare the 4-Tachyon string scattering amplitude in the s-channel with a similar amplitude of 4 scalar particles in the s-channel in quantum field theory. What are the differences in the two cases in the fixed-angle UV limit? What do you expect from the UV behavior of the 4-graviton scattering amplitude in string theory?

(You *do not have to derive* anything in this exercise, just explain your reasoning using the formulae already given in the notes.)