MS-TP526M

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Instituut voor Theoretische Fysica, Universiteit Utrecht

STRING THEORY EXAM

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- The duration of the test is 3 hours.
- Use different sheets for each exercise.
- Write your name and initials on every sheet handed in.
- Write clearly. Unreadable text cannot be judged.
- The lecture notes may be consulted during the test.
- Divide your available time wisely over the exercises.

Problem 1 (*Classical open bosonic strings*)

Classical open bosonic string propagates in 26-dimensional *Minkowski* space-time according to

$$\begin{aligned} X^0 &= A\tau \\ X^1 &= A\cos\sigma\sin\tau\,, \\ X^2 &= A\cos\sigma\cos\tau, \\ X^i &= 0, \quad i \leq 3, \end{aligned}$$

where A is a constant and X^0 is the time direction.

1. Show that the configuration given above satisfies string equations of motion, in particular, the Virasoro constraints.
2. What is the area of the world-sheet swept by string for $\sigma \in [0, \pi]$ and $\tau \in [0, T]$?
3. Consider this solution in the light-cone gauge. Which of the string oscillator modes $\alpha_n^{\pm}, \alpha_n^i$ are excited? What are the values of p^+ and p^-?
4. Compute the mass M^2 and the angular momentum J^{12} corresponding to the rotation in the 12-plane.

Problem 2 (*Counting Virasoro descendants*)

Let $|\Phi\rangle$ be a primary state which is an eigenstate of the number operator N with an eigenvalue N_Φ: $N|\Phi\rangle = N_\Phi|\Phi\rangle$. How many independent Virasoro descendants one has at a fixed level $N_\phi + n$? Motivate your answer.

Problem 3 (*Graviton and dilaton states in covariant quantization*)

Examine the closed string states $\zeta_{\mu\nu}\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}|p\rangle$ with $\zeta_{\mu\nu} = \zeta_{\nu\mu}$.

1. Show that the Virasoro constraints give the conditions $p^2 = 0$ and $p_\mu\zeta^{\mu\nu} = 0$.
2. Exhibit the null states that generate the physical state equivalence $\zeta^{\mu\nu} \sim \zeta^{\mu\nu} + p^\mu\epsilon^\nu + p^\nu\epsilon^\mu$, which holds for $p^2 = 0$ and $p_\mu\epsilon^\mu = 0$.
3. Show that there are $(d-2)(d-1)/2$ independent physical degrees of freedom in $\zeta_{\mu\nu}\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}|p\rangle$ for each value of p_μ which satisfies $p^2 = 0$. These are the degrees of freedom of a graviton and a scalar particle called dilaton.

Problem 4 (*Virasoro primaries*)

Consider the Virasoro operators

$$L_m = \frac{1}{2}\sum_n : \alpha_{m-n}\alpha_n :$$

associated to a *single* open string coordinate X with oscillators that satisfy $[\alpha_m, \alpha_n] = m\delta_{m+n}$. Show that the state

$$|\Psi\rangle = \left(\alpha_{-3}\alpha_{-1} - \frac{3}{4}(\alpha_{-2})^2 - \frac{1}{2}(\alpha_{-1})^4\right)|0\rangle$$

is primary.

Problem 5 (Bonus) *Coherent States*

Consider the following state in the Hilbert space of open bosonic string

$$|\Phi\rangle = e^{v\alpha_{-1}^1 - v^*\alpha_1^1} e^{-iv\alpha_{-1}^2 - iv^*\alpha_1^2}|0\rangle \ ,$$

where v is a complex number.

1. Compute the norm of this state $\langle\Phi|\Phi\rangle$.

2. Compute the expectation value $\langle\Phi|M^2|\Phi\rangle$, where

$$M^2 = \frac{1}{\alpha'}\sum_{n=1}^{\infty}\left(\alpha_{-n}^i\alpha_n^i - 1\right)$$

   is the mass operator for open strings.

3. Choose the parameter v to be $v = v^* = \frac{\sqrt{\pi T}}{2}A$ and take take the light-cone momentum p^+ as

$$p^+ = \frac{1}{\ell}\sqrt{2|v|^2 - 1}\,, \qquad \ell \equiv 1/\sqrt{\pi T}\,.$$

   Compute the space-time expectation value $\langle\Phi|X^\mu|\Phi\rangle$ and show that for large values of $A \geq \ell$ the corresponding expectation value approaches the classical solution described in the Problem 1.

   Hint: You have to recall that in absence of the special momenta p^i the relation between the mass operator and p^- is as follows $p^- = \frac{M^2}{2p^+}$.