Exam Probabilistic Reasoning

4 November 2015, 13:30 – 16:30

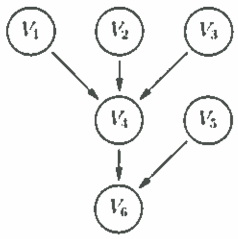
The exam consists of three problems, for each of which the number of credits per question is given. In total, a maximum of 100 credits is awarded. Read the questions very carefully; you may answer them in English and/or Dutch. Be sure to clearly explain your answers!

Good luck!

Reminder: please fill out the Caracal course evaluation. Thanks!

Problem 1 (a: 10 pts., b: 10 pts., c: 15 pts., total: 35 pts.)

Consider a Bayesian network $B = (G, \Gamma)$, where $G = (\mathbf{V}_G, \mathbf{A}_G)$ is the following acyclic digraph and $\Gamma = \{\gamma_{V_i} \mid V_i \in \mathbf{V}_G\}$ is given by:



$\gamma(v_1) = 0.9$ $\qquad$ $\gamma(v_2) = 0.4$

$\gamma(v_3) = 0.5$ $\qquad$ $\gamma(v_5) = 0.5$

$\gamma(v_4 \mid \neg v_1 \wedge \neg v_2 \wedge \neg v_3) = 0.2$ $\qquad$ $\gamma(v_6 \mid v_4 \wedge v_5) = 0.1$
$\gamma(v_4 \mid v_1 \wedge \neg v_2 \wedge \neg v_3) = 0.4$ $\qquad$ $\gamma(v_6 \mid \neg v_4 \wedge v_5) = 0.3$
$\gamma(v_4 \mid \neg v_1 \wedge v_2 \wedge \neg v_3) = 0.6$ $\qquad$ $\gamma(v_6 \mid v_4 \wedge \neg v_5) = 0.7$
$\gamma(v_4 \mid \neg v_1 \wedge \neg v_2 \wedge v_3) = 0.8$ $\qquad$ $\gamma(v_6 \mid \neg v_4 \wedge \neg v_5) = 0$

Variables V_1, V_2 and V_3 have a disjunctive interaction effect on variable V_4. To capture this effect, the assessment function for node V_4 is based on the (leaky) 'noisy-or gate'.

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a. Complete the assessment function $\gamma(V_4)$ for node V_4. Explain your answers.

Let Pr be the probability distribution defined by Bayesian network B. Now, consider the five computation rules of *Pearl*'s data fusion algorithm, given in the attached formula sheet.

b. Consider a node V_c with compound diagnostic parameter $\lambda(V_c) = 1$ for all values of V_c. In addition, consider a parent V_p of V_c. Do we necessarily have for the diagnostic parameters from V_c to V_p that $\lambda_{V_c}^{V_p}(V_p) = 1$ for all values of V_p? Explain your answer.

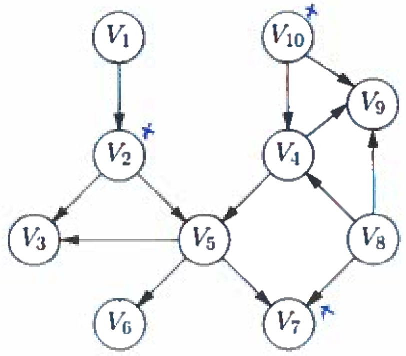
V_p
V_c $\lambda = 1$

A. Yes, this always holds
B. This is only guaranteed in trees, i.e. if V_p is the *only* parent of V_c
C. This is only guaranteed if there is at most one co-parent $V_p' \neq V_p$ of V_c
D. No, this could only happen coincidentally, depending on the actual numbers in the assessment functions.

c. Illustrate Pearl's algorithm by computing the posterior probability $\Pr^{v_6}(v_4)$ from network B.

Clearly indicate which messages/parameters are computed and how; explicitly mention all assumptions you make. Note: you only have to compute messages necessary for establishing the requested probability.

Problem 2 (a: 10 pts., b: 10 pts., c: 10 pts., total: 30 pts.) Consider a Bayesian network $B = (G, \Gamma)$, where $G = (\mathbf{V}_G, \mathbf{A}_G)$ is the following acyclic digraph:



a. Give a loop cutset for graph G that can be found by applying the heuristic *Suermondt & Cooper* algorithm.

b. Give a (minimal or optimal) loop cutset for G that will *not* be found by applying the above mentioned heuristic. Clearly explain why the *Suermondt & Cooper* heuristic will not return this loop cutset. Proof?

Suppose we perform a one-way sensitivity analysis on network B, where we are interested in the output probability distribution $\Pr^{\mathbf{e}}(V_5)$ for variable V_5 given evidence $\mathbf{e}$ for variables $\mathbf{E}$. We want to restrict our analysis to parameters for variables in the *sensitivity set* $S^{\mathbf{E}}(V_5)$ of V_5, i.e. the set of variables whose parameters may upon variation affect $\Pr^{\mathbf{e}}(V_5)$.

c. Suppose the set of observed variables $\mathbf{E}$ consists of V_2, V_7 and V_{10}. Which of the following sets corresponds to the sensitivity set $S^{\mathbf{E}}(V_5)$?

A. $\{V_2, V_4, V_5, V_7, V_8\}$

B. $\{V_2, V_4, V_7, V_8\}$

C. $\{V_4, V_5, V_7, V_8\}$

D. $\{V_4, V_5, V_8\}$

E. none of the above

Clearly explain your answer.

$V_7 \in \sigma^*(V_7)$? think not $\Rightarrow$ D