FINAL SUBATOMIC PHYSICS

20 Apr 2016

For this exam you are allowed to use your book (*Subatomic Physics* by Henley and Garcia), the lecture notes and a simple calculator. Other materials are not allowed.
Start the solution of each exercise on a different sheet of paper.
You may use natural units ($c = \hbar = G = k_B = 1$).
Remember: $c = 3 \cdot 10^8$ m/s and $\hbar = 6.6 \cdot 10^{-16}$ eV/s.

Exercise 1: Energy loss of heavy quarks

Heavy quarks can lose energy via gluon emission (*radiative energy loss*), both in vacuum and in the hot dense medium produced in heavy-ion collisions.

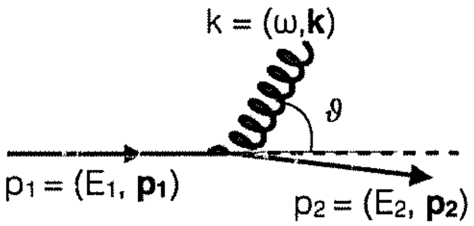

Figure 1: Heavy quark radiating a gluon.

a) Let's first consider the vacuum case, depicted in fig. 1: prove that the angle at which the *virtual* gluon is emitted is given by:

$$\cos\theta = \frac{2E_1\omega - m_g^2c^4}{2\sqrt{(\omega^2 - m_g^2c^4)(E_1^2 - m_q^2c^4)}},$$

where m_g and ω are the mass and the energy of the virtual gluon, m_q and E_1 the mass and the initial energy of the quark.

b) From more detailed QCD calculations of gluon emission probability, it emerges that the radiation is suppressed for $|\theta| < m_q/E_q$ (so-called *dead cone effect*). How would you discriminate between a beauty and a charm quark with an energy of e.g. 10 GeV?
Hint: remember that $m_c = 1.29$ GeV/c^2 and $m_b = 4.2$ GeV/c^2.

In the Quark Gluon Plasma (QGP) produced in heavy-ion collisions, another important mechanism of parton energy loss is through successive elastic collisions with the quarks of the QGP (*collisional energy loss*). The interplay between radiative and collisional energy loss leads, experimentally, to the phenomenon of *jet quenching*.

c) (optional) Do you expect a charm jet, i.e. originating from a charm quark, to be suppressed more or less than a beauty jet, within the QGP?

Suppose now that the propagating parton is a charm quark, which eventually hadronizes in a D^0 meson. The D^0 decays as:

$$D^0 \longrightarrow K^- + \pi^+.$$

d) Work in the center-of-mass frame of the D^0 and express the energy of the kaon as a function of the masses of the three particles and the energy of the pion.

e) Explain how you could experimentally reconstruct this particular decay of the D^{*+} meson:

$$D^{*+} \longrightarrow D^0 + \pi^+.$$

The lifetimes of these particles are $\tau_{D^{*+}} = 7 \cdot 10^{-21}$ s and $\tau_{D^0} = 4 \cdot 10^{-13}$ s. Assume to be able to discriminate a secondary decay vertex[1] from the primary vertex[2] as long as they are spatially separated by ≥ 50 μm, due to limited detector resolution.

Exercise 2: Pentaquark

On 13 July 2015, the LHCb collaboration at CERN reported the observation of a resonance in the decay of bottom Lambda baryons (Λ_b^0), which is consistent with the formation of a pentaquark state. Although its existence was predicted already in the early days of the quark model (Murray Gell-Mann, 1964), it has been experimentally challenging to detect it, up to the point that some physicists were conjecturing new proprieties of quarks that prevented such combinations.
The Λ_b^0 is a neutral baryon containing a bottom quark; it mainly decays as:

$$\Lambda_b^0 \longrightarrow J/\Psi + K^- + p.$$

[1] the point where a secondary particle decays
[2] the point where the first interaction happens, i.e. where the D^{*+} is created

The quark component of each of those particles is reported in table below:

Λ_b^0	b, u, d
J/Ψ	$c, \bar{c}$
K^-	$s, \bar{u}$
p	u, u, d

a) Write down the following quantum numbers of the participants: baryon number (B), strangeness (S), isospin (I_3).
Which forces are involved in this decay?

b) Knowing that the spin states ($|\psi\rangle_{\text{spin}} \equiv |S; S_z\rangle$) of the participants are:

$$|\Lambda_b^0\rangle_{\text{spin}} = |1/2; \pm 1/2\rangle\,, \quad |J/\Psi\rangle_{\text{spin}} = |1; 0, \pm 1\rangle\,, \quad |K^-\rangle_{\text{spin}} = |0; 0\rangle\,,$$

which values of orbital angular momentum ($|\psi\rangle_{\text{orb}} \equiv |L; L_z\rangle$) can the final system ($J/\Psi + K^- + p$) assume? Why? By definition $|\Lambda_b^0\rangle_{\text{orb}} = |0; 0\rangle$.
Hint: remember that the conserved quantum number is always the total angular momentum, $\vec{J} = \vec{L} + \vec{S}$.

c) (optional) Draw the Feynman diagram of such decay.

The pentaquark was discovered observing the invariant mass spectrum of the $J/\Psi + p$ system (fig. 2). Two states were thus identified: $P_c^+(4380)$ and $P_c^+(4450)$, with spin 3/2 and 5/2, respectively[3]. The widths of the resonances are $\Gamma(4380) = 205$ MeV and $\Gamma(4450) = 39$ MeV. The decay chain goes as:

$$\Lambda_b^0 \quad \longrightarrow \quad K^- + P_c^+ \quad \longrightarrow \quad K^- + J/\Psi + p.$$

d) Explain why a "bump" in the invariant mass spectrum of the decay products can be interpreted as a particle being created. What are the quantum numbers of the pentaquark states? What are their lifetimes?

e) (optional) What do you think all the different Λ particles that contribute to the $J/\Psi + p$ invariant mass spectrum are?
N.B.: with the notation Λ, *one indicates that the quark flavor content is fixed to the one of the* Λ_0. *In which quantum numbers can they differ from the latter?*

[3] the number in parentheses refer to the mass, in MeV/c^2

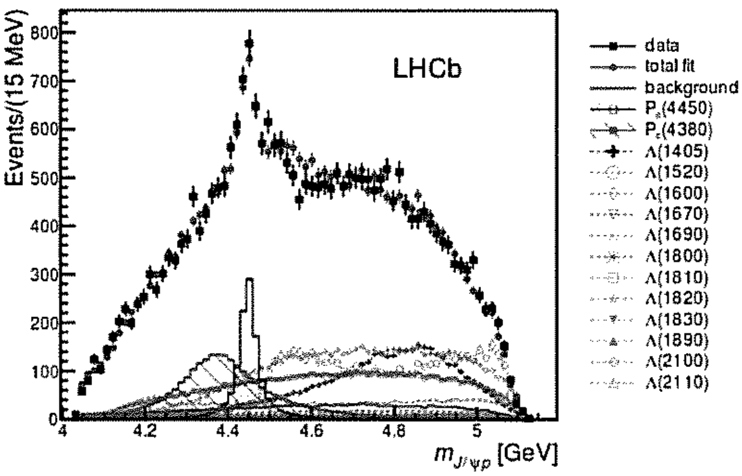

Figure 2: $J/\Psi + p$ invariant mass spectrum. The total fit (red points) is the sum of the components listed in the legend. The contribution of the pentaquarks are shown by hatched histograms.

Consider now the decay of a Λ_b^0 through the pentaquark ground state ($P_c^+(4380)$), so that the final system ($K^- + J/\Psi + p$) will have $S = \frac{3}{2}$ and $L = 1$. Assume the Λ_b^0 to be polarized: $|\Lambda_b^0\rangle_{\text{spin}} = |1/2; +1/2\rangle$.

f) If $\text{P}(|J/\Psi\rangle_{\text{spin}} = |S; S_z\rangle)$ is the probability of finding the J/Ψ in the spin state $|S; S_z\rangle$, compute the ratios:

$$\frac{\text{P}(|J/\Psi\rangle_{\text{spin}} = |1; +1\rangle)}{\text{P}(|J/\Psi\rangle_{\text{spin}} = |1; 0\rangle)}, \quad \frac{\text{P}(|J/\Psi\rangle_{\text{spin}} = |1; -1\rangle)}{\text{P}(|J/\Psi\rangle_{\text{spin}} = |1; 0\rangle)}.$$

Hint: use the Clebsch-Gordan coefficients given in fig. 3.

g) Along the same lines, compute the ratio between the probability that the spins of Λ_b^0 and p are aligned and the probability that they are anti-aligned.

CLEBSCH-GORDAN COEFFICIENTS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

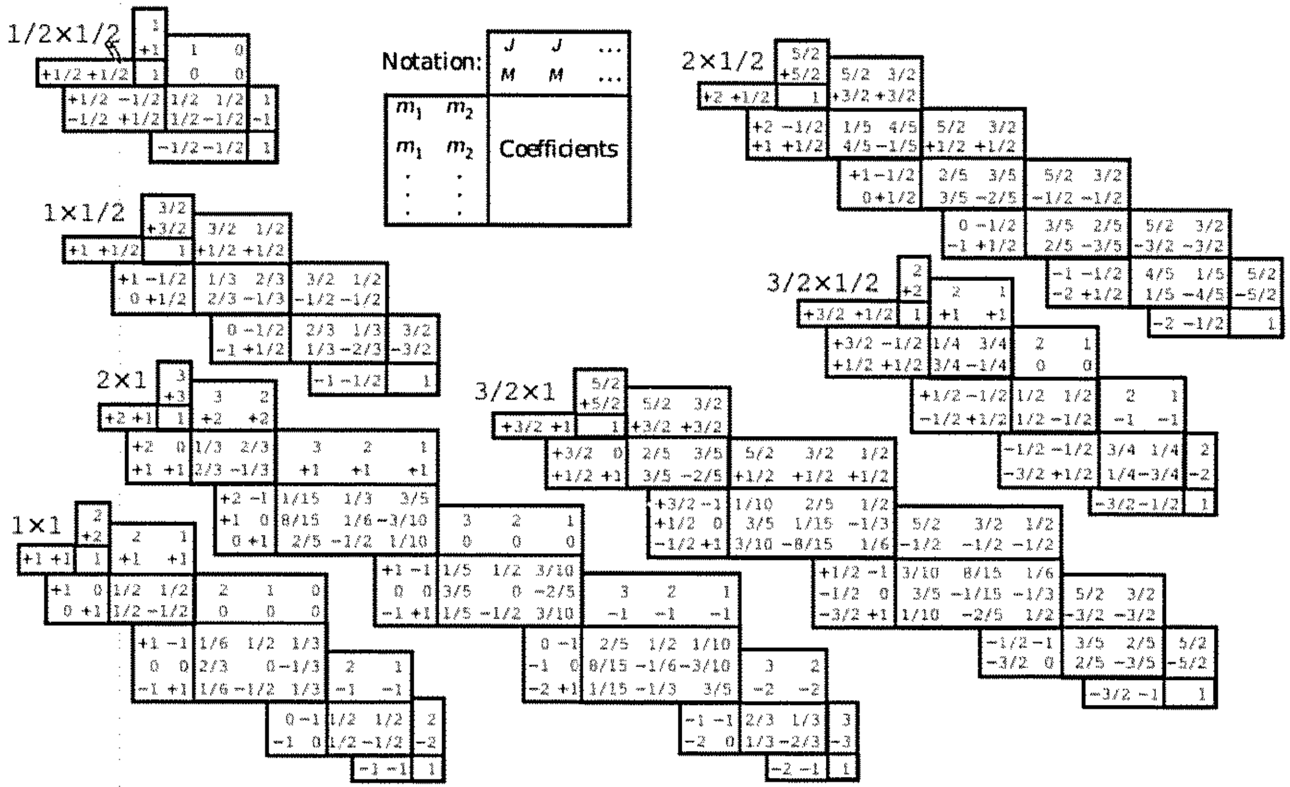

Figure 3: Table of Clebsch-Gordan coefficients. As an example of how to read it: $|0;0\rangle_{ab} = \frac{1}{\sqrt{2}}|1/2;+1/2\rangle_a|1/2;-1/2\rangle_b - \frac{1}{\sqrt{2}}|1/2;-1/2\rangle_a|1/2,+1/2\rangle_b$.