FINAL EXAM SUBATOMIC PHYSICS (June 27th, 2012)

NS-369B

For this exam you are allowed to use your book (Subatomic Physics by Henley and Garcia) and a simple calculator. Other materials are not allowed.

Important note: Start the solution of each exercise on a different sheet of paper. Explain your answers clearly, and don't forget to write units.

Grading: The exam consists of three questions, which were meant to take about equal times to solve. The first two exercises will be more important for your grade than the last one. The point distribution is not fixed yet, but will roughly be (4/4/2).

Exercise 1: linac

The role of accelerators is to deliver a stable, well-focused beam of particles. This exercise is about linear accelerators, or *linacs* (see Fig. 1). For the rest of the exercise assume we have a linac with an accelerating potential V_0, and frequency f.

a) Show that the length of the i-th tube of the linac needs to be $L_i = v_i/2f$, where v_i is the speed of the particle that is accelerated when it is traveling through tube i.

b) Express the speed v_i in terms of the mass of the particle, and its total energy E_i.

c) Suppose we have successfully used the linac to accelerate anti-protons. Can we, without changing anything to the linac, use it to accelerate electrons? Why or why not?
Hints: $m_{\bar{p}} = 938\ MeV/c^2$, $m_e = 511\ keV/c^2$.

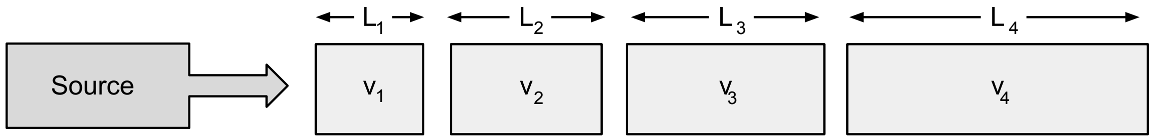

Figure 1: Schematic drawing of a linac, consisting of multiple polarized tubes. The i-th tube has length L_i, and the particles traveling through it have velocity v_i.

Exercise 2: a simple model of the strong nuclear force

In this exercise you will be looking at a simple model of the Strong Nuclear Force. First let us recall the electric force.

a) The electric force, which is responsible for binding electrons to the nucleus, has a potential of the form $V_E(r) \propto -q_1 q_2/r$. Show that the attractive force $F_E(r)$ between two oppositely charged particles goes to zero as $r \to \infty$.

The fact that the electric force becomes negligible at long range makes it possible for electrically charged particles to exist as free particles. Quarks, which have besides an electric charge also a color charge, are never observed as free particles. They exist only as part of a color charge neutral bound state of two (mesons) or three quarks (baryons). We therefore know that the strong force does not become negligible for large r. In fact, numerical QCD calculations show that the potential between a quark q and an anti-quark $\bar{q}$ is well described by:

$$V_S(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr. \tag{1}$$

b) Sketch the potential in eq.(1), and describe what happens to the strong force if one tries to pull apart the two quarks in a meson.

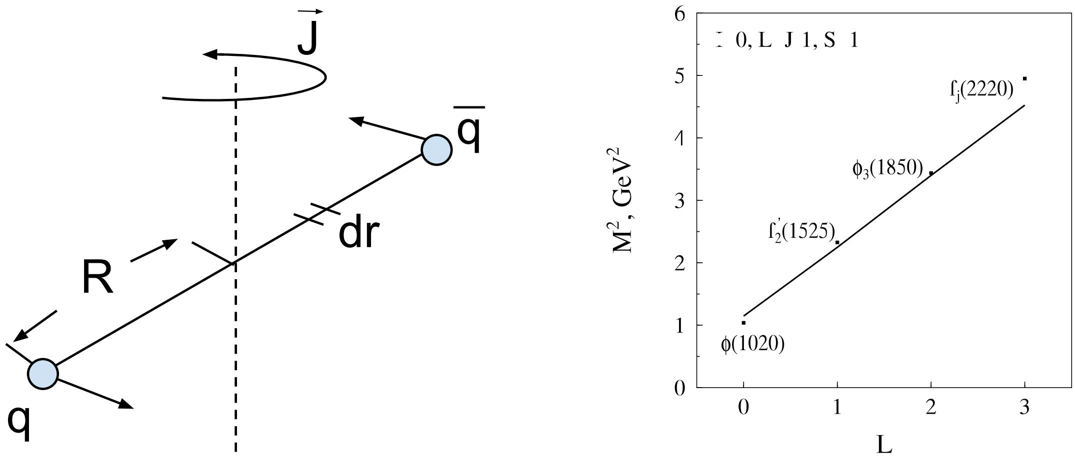

Figure 2: Left: schematic overview of the string model for a meson. Right: M^2 (GeV^2/c^4) as a function of $L = J - 1$ $(\hbar)$ for the ϕ meson and its first excited states.

In principle any term we would add to the strong potential of the form kr^n, $n \geq 1$ would cause the strong force to stay important at long range. As we mentioned, QCD predicts that $n = 1$, however we also have strong experimental evidence for this.

c) Suppose we describe a meson simply as a quark and an anti-quark rotating around one another at a distance $2R$ (see fig.2, left), held together by a string (potential $V(r) = kr$). Both quarks are assumed to be massless, and to travel at the speed of light. Show that the total energy content of the string is $E = \pi k R$.

Hint 1: for a string which is standing still, the energy content in an infinitesimal part dr of the string is simply $dE = kdr$. At relativistic speeds however, one needs to do a Lorentz transformation $dE \to \gamma dE$.

Hint 2: What do you know about the speed of the infinitesimal string element dr as a function of r?

Hint 3: $\int_0^1 1/\sqrt{(1-x^2)}dx = \pi/2$.

d) Now we will calculate the total angular momentum in the string. From classical mechanics you know that for an infinitesimal mass element dm, the infinitesimal contribution to the angular momentum is given by: $d\mathbf{J} = \mathbf{r} \times \mathbf{v}\, dm$. Relativistically, $dm \to dE/c^2$. Show that the total angular momentum of the string is given by $J = \frac{kR^2\pi}{2c}$.

Hint 4: $\int_0^1 x^2/\sqrt{(1-x^2)}dx = \pi/4$.

e) From the previous results deduce that $J = \frac{M^2c^4}{2\pi kc}$, where M is the meson mass, and we have set $E = Mc^2$. Fig.2 (right) shows a measurement of the spin versus the mass squared of the ϕ meson with several of its excited states; deduce a value of k, in units GeV/fm.

Numerical values for the constants: $c = 3.00 \cdot 10^8$ m/s, $\hbar = 6.58 \cdot 10^{-16}$ eV·s

Previous steps showed us that the linear term of the potential is experimentally plausible. This then roughly explains why we do not see free quarks. We can go a step further and use this simple model to explain the mass of hadrons. Consider for istance the ϕ meson: the individual strange and anti-strange quarks have a mass of around 100 MeV/c^2 each, but the ϕ meson itself has a mass of around 1.02 GeV/c^2.

f) Suppose now that the *extra* mass of the ϕ meson (the one which does not come from the quarks) is generated by the energy in the strong force field. Based on the constant k you found in the previous exercise, make an estimate of the radius of the ϕ meson. (use $k = 1$ GeV/c if you did not manage to find it).