FINAL EXAM SUBATOMIC PHYSICS - SOLUTIONS (June 27th, 2012)

NS-369B

Exercise 1: linac

a) If T_i is the time that the particle is in tube i, then $v_i T_i = L_i$. Suppose e^- particles are accelerated, and the electrons are currently traveling through tube i, then by the time the particle reaches the end of tube i, the polarity of tube i must be $-$, while the polarity of tube $(i+1)$ must be $+$. By the time the end of tube $(i+1)$ is reached, the polarity must have switched, i.e., half an oscillation has passed, i.e., a time $T_i = 1/2f$ has passed. Hence $L_i = v_i/2f$.

b) Using $E_i = \gamma_i mc^2$, we find that $v_i = c\sqrt{1 - m^2c^4/E_i^2}$.

c) From the previous two questions we can see that $2fL_i/c = \sqrt{1 - m^2c^4/E_i^2}$, where the l.h.s. is assumed to be constant. This means that also the r.h.s. needs to be constant, i.e., the ratio m/E_i needs to be constant.

Now assume that the particle comes from the source with an energy E_0, then in the i-th tube, it has energy $E_i = E_0 + iV_0$, which is also assumed to be constant.

In conclusion, one cannot simply use the same linac, but both E_0 and V_0 need to be changed such that the ratio m/E_i is the same for all i.

Exercise 2: a simple model of the strong nuclear force.

a) For the force we have:

$$\mathbf{F}_E(r) = -\nabla V_E(r) = \frac{q_1 q_2}{r^2}\hat{\mathbf{r}},$$

which vanishes for $r \to \infty$.

b) The force will become constant ($\mathbf{F}_S(r) = -k\hat{\mathbf{r}}$) at large distances.

c) The total energy in the string can be found by integrating:

$$E = \int_{-R}^{R} dE \to 2k\int_0^R \gamma dr = 2k\int_0^R \frac{dr}{\sqrt{1-\frac{v^2}{c^2}}}$$

However, since the quarks at the end points of the string go with the speed of light, we must have: $v(r) = cr/R$. Hence it follows:

$$E = 2k\int_0^R \frac{dr}{\sqrt{1-\frac{r^2}{R^2}}} = 2kR\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \pi kR,$$

where we've used $x = rR$.

d) In a very similar way we have:

$$J = \int_{-R}^{R} dJ = \frac{2}{c^2}\int_0^R rvdE = \frac{2k}{c^2}\int_0^R \gamma rvdr = \frac{2k}{c^2}\int_0^R \frac{rvdr}{\sqrt{1-\frac{v^2}{c^2}}}$$

Now again we substitute $v(r) = cr/R$, followed by the coordinate transformation $x = rR$. We get:

$$J = \frac{2k}{cR}\int_0^R \frac{r^2dr}{\sqrt{1-\frac{r^2}{R^2}}} = \frac{2kR^2}{c}\int_0^1 \frac{xdx}{\sqrt{1-x^2}} = \frac{kR^2\pi}{2c}.$$

e) Using the results of (c) and (d) we can eliminate R: $M^2c^4 = E^2 = \pi^2k^2R^2 = 2Jc\pi k$. Hence we find: $J = \frac{M^2c^4}{2c\pi k}$. From the graph we can roughly deduce the slope of the line. Comparing the ground state and the highest excited state in the graph we find: $\Delta J = 3\hbar$, $\Delta M^2c^4 = 4$ GeV^2. For the natural constants we have: $3.00 \cdot 10^{23}$ fm/c, and $\hbar = 6.58 \cdot 10^{-25}$ GeV·s.

Plugging in the numbers we thus find:

$$k = \frac{1}{2\pi c\hbar}\frac{\Delta M^2c^4}{\Delta J} = \frac{1}{2\pi(3.00 \cdot 10^{23} fm/s) \cdot (6.58 \cdot 10^{-25} GeV \cdot s)}\frac{4}{3}GeV^2 \approx 1GeV/fm.$$

f) Hence for the size of the meson we find $m_\phi - 2m_s = \pi kR \to R \approx 0.3$ fm.