Group theory – Exam

Notes:

1. **Write your name and student number *****clearly***** on each page of written solutions you hand in.**
2. You can give solutions in English or Dutch.
3. You are expected to explain your answers.
4. You are **not** allowed to consult any text book, class notes, colleagues, calculators, computers etc.
5. Advice: read all questions first, then start solving the ones you already know how to solve or have good idea on the steps to find a solution. After you have finished the ones you found easier, tackle the harder ones.

1) Compute the center of D_n for $n \geq 3$. Analyse carefully the cases n even and n odd.

2) For each list of groups a), b) and c) below, decide which of the groups within each list are isomorphic, if any:

a) $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_9 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_{18} \times \mathbb{Z}_2$ and $\mathbb{Z}_6 \times \mathbb{Z}_6$.

b) S_4, $A_4 \times \mathbb{Z}_2$, D_{12} and $\mathbb{H} \times \mathbb{Z}_3$, where $\mathbb{H}$ is the quaternion group with 8 elements.

c) $(\mathbb{Q}, +)$, $(\mathbb{R}, +)$, $(\mathbb{R}_+, \times)$.

3) Show that $(\mathbb{Q}, +)$ is not finitely generated, i.e., if $X \subset \mathbb{Q}$ is a finite set, then the group generated by X is not $\mathbb{Q}$.

4) Let p be a prime and X be a set with less than p elements. Show that the only action of $\mathbb{Z}_p$ on X is the trivial one.

5) Let G be a finite group and $H < G$ be a subgroup of index n, i.e., $\#G = n\#H$. Show that $g^{n!} \in H$ for every $g \in G$.

6) Show that if a group G has a conjugacy class with two elements then G is not simple.