Group theory – Exam 2

Notes:

1. **Write your name and student number ***clearly*** on each page of written solutions you hand in.**
2. You can give solutions in English or Dutch.
3. You are expected to explain your answers.
4. You are **not** allowed to consult any text book, class notes, colleagues, calculators, computers etc.
5. If you are not sure about some definition of notation you encounter in the exam, please ask.
6. Advice: read all questions first, then start solving the ones you already know how to solve or have good idea on the steps to find a solution. After you have finished the ones you found easier, tackle the harder ones.

1) Let H be a subgroup of a group G and K be a normal subgroup of G. Show that $H \cap K$ is a normal subgroup of H.

2) For each list of groups below, decide which groups are isomorphic, if any:

a) $D_9 \times \mathbb{Z}_2$, D_{18} and $D_6 \times \mathbb{Z}_3$, .

b) $D_{12} \times \mathbb{Z}_2$, D_{24}, $D_8 \times D_3$ and $S_4 \times \mathbb{Z}_2$.

3) Let G and H be groups show that $G \cong G \times \{e\} \subset G \times H$ is a normal subgroup of $G \times H$ and that the quotient $G \times H/G$ is isomorphic to H.

4) Classify all groups of order $7^2 \cdot 17^2$.

5) Given a group G, we define a sequence of groups by induction setting $G_0 = G$ and $G_n = G_{n-1}/Z_{G_{n-1}}$.

a) Show that if G is Abelian, then $G_i = \{e\}$ for $i > 0$;

b) Show that if G is simple and not Abelian, then $G_i = G$ for all i;

c) Compute this sequence for A_5, $\mathbb{Z}_{10}$, D_{10} and D_8.