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Eerste deeltentamen Speltheorie (WISB272)
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Bij dit tentamen is het boek 'Game Theory' van H. Peters toegestaan.

Opgave 1

In a far galaxy, a war is raging between Count Xavier and General Yu. The fighting is taking place at two locations, the planets Alpha Proxima and Vega. Xavier has four starships available, which he can distribute over the two locations in three different ways: one at Alpha Proxima and three at Vega, two at each planet or three at Alpha Proxima and one at Vega. Yu only has three starships, which he can distribute in two ways: one at Vega and the other two at Alpha Proxima, or two at Vega and one at Alpha Proxima. If Xavier sends m_1 starships to Alpha Proxima and Yu sends n_1 starships there, then the payoff for both Xavier and Yu is zero when $m_1 = n_1$. If $m_1 > n_1$, then the payoff to Xavier is n_1 and to Yu it is $-n_1$. If $n_1 > m_1$, the payoff to Xavier is $-m_1$ and to Yu it is m_1. A similar rule applies at the location Vega. Each players payoff is equal to the sum of the payoffs at the two sites.

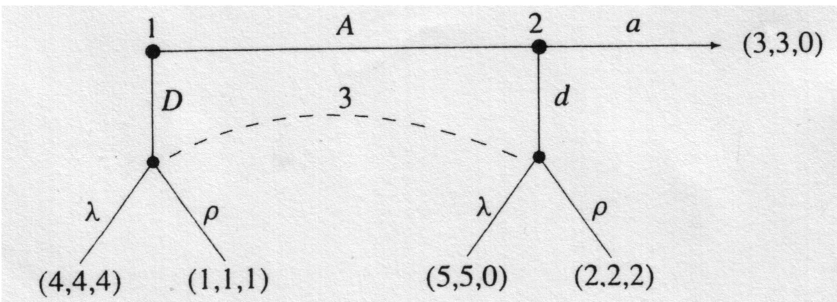
a) Argue that this is a zero-sum game and set up the strategic form.

b) Determine the value of the game

c) Find the optimal strategies for both players.

Opgave 2

Consider the following three-person game of incomplete information (this game is known as 'Selten's horse'):



a) Prove that (A, a, ρ) and (D, a, λ) are both pure-strategy Nash equilibria.
Use the concept of best reply rather than the strategic form of the game. Which, if any, of these solutions are subgame perfect?

b) Which, if any, of these solutions is a perfect Bayesian equilibrium? If you identify a perfect Bayesian equilibrium, also give the corresponding beliefs of player 3.

Opgave 3

Consider the following bimatrix game:

$$\begin{pmatrix} (1,1) & (4,2) & (5,3) \\ (2,5) & (5,3) & (2,3) \\ (3,2) & (2,1) & (4,2) \end{pmatrix}$$

a) Eliminate any strategy that is strictly dominated by other strategies, pure or mixed. Repeat this procedure, if possible.

b) Find all Nash equilibria of the game.

Opgave 4

Two firms are developing competing products for a market of fixed size. The longer a firm spends on development, the better the product. But the first firm to enter the market has an advantage: the customers it obtains will not subsequently switch to its rival.
A firm that releases its product first, at time t, captures a share $h(t)$ of the market, leaving the rest to its competitor. If both release at the same time, both get one half of the market. $h(t)$ is a strictly increasing continuous function on the interval $[0, T]$, where T is some fixed time. It has the properties that $h(0) = 0$ and $h(T) = 1$. Both firms want to maximize their market share and are therefore confronted with the question at what time to release their product.

a) For each firm, write down the strategy set and the payoff function.

b) Find all Nash equilibria of this game, if they exist.

Opgave 5

The IT-company Volks-soft wants to take over its competitor Universal Solutions, who would rather stay independent. A well known tactic in such a situation is for Universal to go into debt, for instance by borrowing from a bank and investing in new buildings or hardware. This makes Universal less attractive to Volks-soft. This tactic of Universal is known as the 'poison-pill defense'.
However, if Universal is financially weak, the burden of extra interest payments to the bank outweighs the benefit of the investments. If Universal is financially strong, the investment will be rewarding for Universal.
Volks-soft does not know whether Universal is financially strong or not, but knows that Universal is strong with probability 0.5, and weak also with probability 0.5. The game is played as follows. First, Chance decides whether Universal is strong or weak, according to the probabilities mentioned above. Then Universal moves, by either Investing (I) or Not Investing (N). If Universal is strong, playing I will give it a payoff of +1, if it is weak a payoff of -1. Playing N gives a payoff to Universal of 0.
After observing Universal?s move, Volks-soft then decides to Takeover (T) or Pass (P). If Universal has not invested, the payoff to Volks-soft is +2 if it takes over a strong company and -1 if it takes over a weak company. If Universal has invested, the payoffs to Volks-soft after playing T are +1 if Universal is strong and -2 if Universal is weak. Playing P always gives a payoff of 0 for Volks-soft. Finally, Universal gets a bonus of +1 if it is not taken over.
All of the above is common knowledge and both firms want to maximize their profits.

a) Determine the extensive form of this signaling game.

b) Find all the pure Nash equilibria of this game.

c) Which of these equilibria are perfect Bayesian? You only need to give the beliefs in the relevant information set. Which equilibria are pooling and which are separating?