DEPARTEMENT WISKUNDE, FACULTEIT BÈTAWETENSCHAPPEN, UU.
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Tweede deeltentamen Speltheorie (WISB272)
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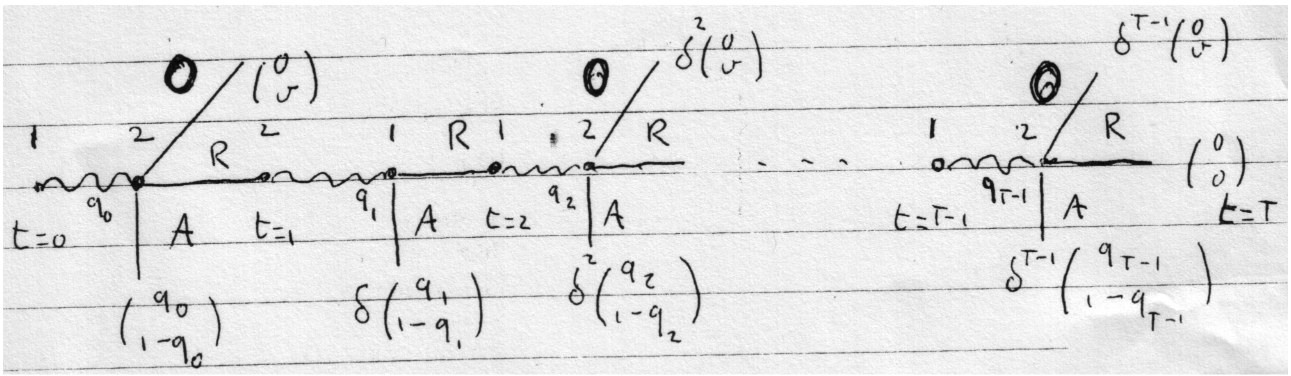
Opgave 1

Consider the following variation of the sequential (Rubinstein) bargaining game. As in the standard version, two players alternate proposing a split of a certain good. The total amount to be divided shrinks with each time step by a factor $0 < \delta < 1$. In this case, when player 2 moves after an offer by player 1 (that is, at time t with t even), he has three instead of two options. As usual, he can Accept player 1's offer, he can Reject it and propose an alternative split, or he can play his Outside option. If he chooses O, he will receive a share $\delta^t v$ of the good and player 1 will receive nothing. In many bargaining situations this is a real possibility. Player 2 may know of a third party who guarantees delivery of $\delta^t v$, and if player 2 chooses this option, player 1 is left empty-handed. The extensive form for the finite-horizon version is given below. In this case, the final time $t = T$ is odd (in Dutch: 'oneven').



a) Take $T = 1$. Show that the following strategy:
   player 1: at $t = 0$, offer $q_0 = 1 - v$ (S_1)
   player 2: at $t = 0$, accept any offer $q_0 \leq 1?v$
   at $t = 0$, choose O if $q_0 > 1 - v$ (S_2)
   is a subgame perfect equilibrium (SPE).

b) Show that player 1 does not have a best reply to the alternative strategy
   player 2: at $t = 0$, accept any offer $q_0 < 1 - v$
   at $t = 0$, choose O if $q_0 \geq 1 - v$ (S_2^*)
   Use this to prove that the set S_1, S_2 is the unique SPE for the game with $T = 1$.

c) Take $T = 2$. Give the diagram of the extensive form. Describe the SPE, by giving strategies of players 1 and 2 at $t = 0$ and $t = 1$. Do this for the two cases $\delta > v$ and $\delta < v$.
   Give your answers in the form indicated below:
   player 1: at $t = 0$, offer $q_0 = \ldots$
   at $t = 1$, accept if $\ldots$, reject if $\ldots$
   player 2: at $t = 0$, accept if $\ldots$, reject if $\ldots$, play O if $\ldots$
   at $t = 1$, offer $q_1 = \ldots$
   Motivate your answer, preferably with diagrams.

d) From now we will consider the infinite horizon version of this game. Let x^* be the split offered by player 1 in the SPE of the game without outside option, see page 89-90 of Peters. Assume

$v < x_2^*$. Formulate the SPE for the game with outside option. Motivate your answer. You may use the fact that the strategy set σ_1^*, σ_2^*, defined on pages 89-90 of Peters, is an SPE of the game without outside option.

e) Assume $v > x_2^*$. Show that the following strategy pair is a SPE.

player 1:	always propose $(1-v, v)$. Accept (z_1, z_2) if and only if $z_1 \geq \delta(1-v)$.
player 2:	always propose $(\delta(1-v), 1-\delta(1-v))$. Accept (z_1, z_2) if $z_2 \geq v$. Play O if $z_2 < v$. Never reject.

Describe the outcome of the game and the payoffs.

f) We will now consider a more general bargaining game, where player 1 has utility function $u_1(\alpha) = \alpha$ and player 2 has utility $u_2(\alpha) = \sqrt{1-(1-\alpha)^3}$.
Assume there is no outside option. Sketch the feasible set in the (u_1, u_2)-plane.

g) Still assuming no outside option, use 10.1 on page 137 of Peters to find expressions for the subgame perfect offers $x^*(\delta)$ and $y^*(\delta)$. Please note that page 137 contains an annoying misprint. The third line from the bottom should read: '...we obtain $y_1 = \delta\alpha$ and thus $\sqrt{1-\alpha} = x_2 = \delta y_2 = \delta\sqrt{1-\delta\alpha}$.' Note the extra δ in the last expression.

h) Calculate $\bar{x} = lim_{\delta \to 1} x^*(\delta)$ to obtain the bargaining solution. Also give the? distribution in terms of utilities.
Hint: $1-\delta^n = (1-\delta)(1+\delta+\cdots+\delta^{n?1})$.

i) Now assume that player 2 has an outside option $v > \bar{x}_2$. Give the coordinates of the outcome of the SPE, as $\delta \to 1$, in the (u_1, u_2)-plane and indicate the location of this bargaining solution in your sketch in (f).

Opgave 2

The Stag Hunt game takes it's name from a story by the French philosopher Jean Jacques Rousseau. Two players go hunting. If they work together, they can catch a stag (male deer). If each hunts on his own, he can only catch a hare. The value of half a stag is more than the value of a whole hare, so the payoff for player 1 can be given by:

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Here, the first row (column) represents the choice 'hunt together' for player 1 (player 2) and the second row (column) represents the choice 'hunt alone' for player 1 (player 2). The payoff table for player 2 is A^T (the transposed of A), so this is a symmetric game.

a) Find all Nash equilibria

b) Determine which of these are ESS (evolutionary stable strategies). Prove your statements.

c) Derive the corresponding replicator equation. Calculate the fixed points. Sketch the phase diagram and indicate which fixed points are stable.

Opgave 3

Let the TU game (N, ν) be given by: $N = 1, 2, 3, \nu(1, 3) = \nu(N) = 1$, $\nu(1, 2) = \beta < 1$ and $\nu(S) = 0$ for all other coalitions S.

a) Determine the core of this game.

b) Determine the Shapley value of this game. Is there a value of β for which the Shapley value is in the core?

c) Consider the 'weighted majority game', given by $N = 1, 2, 3, \nu(S) = 1$ if $W(S) \geq 3$ and $\nu(S) = 0$ otherwise. Here, $W(S) = \sum_{i \in S} w_i$ with $w = (1, 1, 2)$. Calculate the nucleolus of this game.