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First Exam Game Theory (WISB272)
November 4, 2009

The use of 'Game Theory', by H. Peters is allowed. You can write your answers in Dutch.

Question 1.

From "Perfect Foresight and Economic Equilibrium" by O. Morgenstern 1935, based on the short story "The final problem", by A. Conan Doyle, 1893.

Sherlock Holmes is pursued (achtervolgd) by his enemy, professor Moriarty. Holmes tries to escape by taking a train from Victoria Street station to Dover. This train has one intermediate stop at Canterbury. As the train leaves, Holmes spots Moriarty on the platform and also realises that Moriarty has spotted him as well.
Moriarty has two choices: take a fast train to Canterbury or take a fast train to Dover. In both cases, Moriarty will arrive before Holmes. Holmes has the option of getting off at Canterbury, or continuing all the way to Dover.
If Holmes gets off at a station where Moriarty has already arrived, he will be greatly harmed and possibly killed by Moriarty. This outcome is represented by a payoff of +100 for Moriarty and -100 for Holmes. If Holmes arrives at Dover, while Moriarty has taken the fast train to Canterbury, Holmes escapes unharmed to the Continent. This outcome is represented by a payoff of -50 for Moriarty and +50 for Holmes. Finally, if Holmes gets off at Canterbury while Moriarty has taken the fast express to Dover, Holmes is safe for the moment. This outcome is represented by a payoff of 0 for Moriarty and 0 for Holmes.

a) Write down the strategic form of this zero-sum game.

b) Determine the value of the game.

c) Find the optimal strategies for both players.

P.S.: in the story, Holmes gets off at Canterbury, while Moriarty takes the fast train to Dover. Unfortunately, the evil professor later catches up with the great detective at Reichenbach Falls...

Question 2.

Auction the Dollar is a game where an amount of money is auctioned (!), but this auction has some special rules. This auction game can be profitable for organisers. Try it at home! Its inventor, Martin Kubik, remarks: "the best time to play is during a party when spirits are high and the propensity to calculate has declined". Below is a simplified version.
An amount of v euro's will be auctioned amongst two people. They take turns in bidding. A bid must be a positive integer and it must be strictly larger than the bid of the previous player. A player may never bid more than his wealth w.
The game is over when one of the two players plays "pass". When the game is over, the player who did NOT pass receives the prize v.
However, both players must pay the organiser their last positive bid. For instance, if player one starts with "pass" the game is over and player two receives v and player one receives 0 (zero). If player one starts with bidding 2, player two then bids 5 and player one then passes, the payoffs are -2 to player one and $v - 5$ for player two. Also note that if the previous player has bid w, the current player has no other option but to play "pass".

a) Let $v = 2,5$ and $w = 3$. Draw the extensive form of this game and find all subgame perfect equilibria.

We now change the rules as follows. Each player makes only one bid. The first player makes a positive integer bid less or equal to w. The second player is not told the bid of the first player. The second player can play "pass" or any integer bid less or equal to w. The highest bid wins the prize, but both players also have to pay their bid to the organiser. In case of a tie, the first player wins.

a) Again, let $v = 2,5$ and $w = 3$. Write down the strategic form of this game.

b) Find all Nash equilibria. Start by eliminating dominated strategies. Also calculate for every equilibrium the expected payoff for the first player and for the second player.

Question 3.

A manufacturer of bicycle tires has a production cost of 4 euro's per tire. It sells its tires to a retailer (winkel), who in turn sells it to the consumers. Assume that the market price p as function of q, the quantity of tires in the market, is given by the relation $p = 12 - q$, where q is measured in some appropriate unit. The manufacturer sets a price x, which the retailer has to pay to the manufacturer for each tire. Then, the retailer decides how many tires he will buy. The manufacturers payoff is $q(x - 4)$ and the payoff for the retailer is $q(p - x)$.

a) Write down the strategy sets of the manufacturer and the retailer and their payoff functions. Find the equilibrium levels q^* and x^* that correspond with the Nash equilibrium of this game.

b) Now assume there is no retailer and that the manufacturer sells directly to the consumer. Calculate the equilibrium quantity for this case.

c) Compare the pricelevels and profits of the situation with and without a retailer. What overall effect does the introduction of a retailer in a monopoly market have for consumer, manufacturer and retailer?

Question 4.

Paulina and Gretchen live outside a city with one cinema. Gretchen wants to go to the movies and she doesn't mind what movie is playing. Her friend Paulina only wants to go if "A Beautiful Mind" is playing, which happens to be 50% of the time. Gretchen, who lives above a cigarshop, can buy a newspaper for 2 units. In this newspaper, the movie that is shown that night is listed.
Gretchen has a boyfriend, Diego, who works at the cinema and who tells Gretchen what the movie of the night is. Diego always tells the truth, but Paulina doesn't trust Diego (it all has to do with that weekend on the beach, but that's another story). Paulina only trusts the newspaper. Since Paulina owns and drives the car, she finally decides whether she and Gretchen go (G) to the cinema or stay home (S).
If they go to the cinema and "A Beautiful Mind" is playing, Paulina derives a utility worth 1 unit and Gretchen has a utility worth 3 units. If another movie is playing, Paulina derives a utility worth -1 unit (she's grumpy) and Gretchen has a utility worth 1 units (she has to put up with grumpy Paulina, but can enjoy a movie, and see Diego). If Paulina decides they are not going, both receive a utility of O.
So the game starts with a move by Nature, namely whether "ABM" is playing (P) or some other movie (O), both events occurring with probability 1/2. Thanks to Diego, Gretchen knows the result. Then Gretchen has to decide whether to buy (B) a newspaper to show Paulina, or not (N). Finally, Paulina has to choose between G and S.
If Gretchen buys a newspaper, this cost has to be substracted from her utilities.

a) Determine the extensive form of this game.

b) Note that if Gretchen plays "B", then Paulina has full information. Argue that Gretchen's strategy set has four elements, whereas Paulina's strategy set has only two elements. Write down the strategic form of this game and find all the pure Nash equilibria.

c) Which of these equilibria are perfect Bayesian? Give the corresponding beliefs. Which equilibria are pooling and which are separating?