Institute of Mathematics, Faculty of Mathematics and Computer Science, UU.
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In 2003/2004, the course WISB272 was given by Alexander Gnedin.

Speltheorie (hertentamen) (WISB272)
21 augustus 2003

Schrijf je naam en registratienummer op vel no. 1 en schrijf je naam ook bovenaan elk ander in te leveren vel.

Question 1

Consider a version of Nim played exactly as Nim except that in each move a player may take away any number of stones from one or two heaps. There are n heaps and at least one stone must be taken in a move. Each position $(k_1, \ldots, k_n)$ can be represented as a rectangular matrix with elements 0 and 1 in n rows, such that row j is the expansion of k_j in base 2, perhaps complemented by some zeroes on the left.

a) Prove that a position is a P-position if and only if the number of 1's in each column of the matrix is divisible by 3.

b) Use the result in (a) to decide if $(12, 19, 27)$ is a N-position and if 'yes' find the first winning move.

Question 2

A game on a 4×4 chessboard with two rooks is played by two players. The rooks may occupy the same or different squares. As the game starts, both rooks are on the square $(1, 1)$, and in each move a player may select one of the rooks to move. If the player decides to move a rook in square (x, y) she must move it to either a square (x', y) with $x' > x$ or a square (x, y') with $y' > y$. The players alternate the moves and the last player to move wins.

a) Decide if the initial position is a P-position.

b) Determine the Sprague-Grundy function of the game.

Question 3

Solve the game with matrix

$$\begin{pmatrix} 2 & 5 & 1 & 7 \\ 4 & 3 & 6 & 2 \end{pmatrix}$$

Question 4

Find all Nash equilibria in the following games:

a) $\begin{pmatrix} 0,0 & -2,1 \\ 1,-2 & -3,-3 \end{pmatrix}$

b) $\begin{pmatrix} 1,1 & 3,3 \\ 4,0 & 0,4 \end{pmatrix}$

c) $\begin{pmatrix} 0,0 & 0,0 & 2,2 \\ 0,0 & 3,3 & 0,0 \\ 1,1 & 0,0 & 0,0 \end{pmatrix}$.

Question 5

Consider the cooperative game with bimatrix $\begin{pmatrix} 1,2 & 4,3 & 6,2 \\ 4,0 & 3,3 & 2,2 \end{pmatrix}$.

a) Assuming nontransferable utility, sketch the feasible set and determine its Pareto boundary.

b) Assuming a transferable utility find the TU-solution, including threat strategies, disagreement point and side payment.

Question 6

Consider the three-person game in coalitional form with the characteristic function $v(\emptyset) = 0$, $v(\{1\}) = 0$, $v(\{2\}) = 1$, $v(\{3\}) = -1$, $v(\{1,2\}) = 2$, $v(\{1,3\}) = 0$, $v(\{2,3\}) = 1$, $v(\{1,2,3\}) = 3$.

a) Decide if the core is empty and if 'no' find a stable imputation.

b) Compute the Shapley value.

Question 7

Consider the Cournot duopoly model with price per item equal to $(29 - s)_+ + 1$ where s is the total amount produced. Producer's i cost of producing $x \geq 0$ items is $x + i$.

a) Suppose there are two producers. What is their equilibrium production and the total profit?

b) Suppose there are n producers. Find all possible values of n such that the equilibrium production is profitable for each of the producers.