Exam for *Complexe Functies*.

A. Henriques, June 2009.

Problem 1 Write down a power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ that has a double zero at the origin, and that has radius of convergence equal to π.

Problem 2 Prove that if both $f(z)$ and $\overline{f(z)}$ are holomorphic, then f is a constant function.

Problem 3 Let $\gamma : [0, \pi/2] \to \mathbb{C}$ be the path given by $\gamma(t) := e^{it}$. Draw the path γ. Find a function f such that

$$\int_\gamma f(z)dz = 1.$$

Problem 4 Let $a > 1$ be a real number. Compute the integral

$$\int_{-\pi}^{\pi} \frac{1}{a + \cos(t)}\, dt.$$

Problem 5 Compute the Laurent expansion of $\frac{1}{z^2 + z^4}$ in the domain $\{z \,:\, |z| > 1\}$.

Problem 6 Prove that integrals along paths are left invariant under orientation preserving reparametrizations, and are negated by orientation reversing reparametrizations.

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