ENDTERM COMPLEX FUNCTIONS

JUNE 26 2013, 9:00-12:00

- Put your name and student number on every sheet you hand in.
- When you use a theorem, show that the conditions are met.

Exercise 1 (*10 pt*) Give an analytic isomorphism between the first quadrant

$$Q = \{z \in \mathbb{C} : \operatorname{Re}(z) > 0 \ \text{ and } \ \operatorname{Im}(z) > 0\}$$

and the open unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$.

Exercise 2 (*25 pt*) Let $a, b > 0$. Prove that the following integrals converge and evaluate them.

a. (*10 pt*) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos(ax) - \cos(bx)}{x^2}\, dx$

b. (*15 pt*) $\displaystyle\int_{-\infty}^{\infty} e^{-ax^2} \cos(bx)\, dx$ (*Hint*: Use a rectangular countour.)

Exercise 3 (*10 pt*) Consider the polynomial function $P(z) = z^7 - 2z - 5$.

a. (*7 pt*) Determine the number of roots of P with $\operatorname{Re}(z) > 0$.

b. (*3 pt*) How many of them are simple?

Bonus Exercise (*15 pt*) Prove that

$$\int_0^{\infty} \frac{\sin(x)}{\log^2(x) + \frac{\pi^2}{4}}\, dx = \frac{2}{e} + \frac{2}{\pi} \int_0^{\infty} \frac{\log(x)\cos(x)}{\log^2(x) + \frac{\pi^2}{4}}\, dx\ .$$

You may assume that the integrals converge.