3RD EXAM 'INLEIDING IN DE GETALTHEORIE'

Tuesday, 25th October 2016, 9 am - 10 am

Question 1
Find the continued fraction of $\sqrt{12}$ and $\sqrt{17}$.

Question 2
Find the quadratic numbers that belong to the continued fractions

$$[3, 1, 6, 1, 6, 1, 6, 1, 6, ...] \quad \text{and} \quad [2, 1, 8, 1, 8, 1, 8, 1, 8, ...]$$

Question 3
Show that for any natural numbers $p, q \in \mathbb{N}$ one has

$$\left|\sqrt{5} - \frac{p}{q}\right| > \frac{1}{5q^2}.$$

Question 4
Find at least two different solutions to the equation

$$1 + 2 + \ldots + k = (k + 1) + (k + 2) + \ldots + (l - 1) + l,$$

with $k, l \in \mathbb{N}$ and $l > k$ and show how it is related to a Pell's equation.

Note: A simple non-programmable calculator is allowed for the exam.

Date: 25th October 2016.