1ST EXAM 'INLEIDING IN DE GETALTHEORIE'

Tuesday, 25th September 2018, 9 am - 10 am

Question 1

Solve the basket of eggs problem: find the smallest number of eggs such that one egg remains when eggs are removed 2, 3, 4, 5, 6 at a time, but no eggs remain if they are removed 7 at a time.

Question 2

Let n be a natural number. We say that n is 5th power free if there is no integer $d \geq 2$ with $d^5 \mid n$. Show that there are arbitrarily long intervals such that no integer in such an interval is 5th power free. I.e. show that for every positive number x there is an interval $[a, b]$ of length x such that none of the integers in the interval $[a, b]$ is 5th power free.

Question 3

Let $n > 1$ be a natural number and a an integer. Assume that either $a > 2$ or that $a = 2$ and n is not prime. Deduce that $a^n - 1$ is not prime.

Question 4

Let $n \geq 1$ be an integer and write $d(n)$ for the number of positive divisors of n. Show that

$$\prod_{t \mid n} t = n^{d(n)/2},$$

where the product is taken over all positive divisors of n.

Note: A simple non-programmable calculator is allowed for the exam.

Date: 25th September 2018.