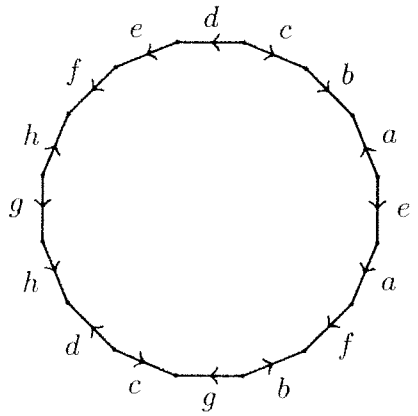
Midterm exam *Topologie en Meetkunde* (WISB341). A. Henriques, Mar 2012.
Do not simply provide answers: justify all your assertions.

Problem 1 State the definition of a manifold. [2pt] [1pt]
Prove that if M and N are manifolds, then their product $M \times N$ is also a manifold. [1pt]

Problem 2 State the classification theorem for compact surfaces. [3pt] [1pt]
Let Σ be the surface obtained by glueing the sides of a regular 16-gon according to the following pattern:



To which surface in the classification is Σ homeomorphic? [2pt]

Problem 3 Given two natural numbers $m < n$, the product $S^m \times S^n$ of the m-dimensional sphere with the n-dimensional sphere is a CW-complex with four cells. [3pt]
What are the dimensions of those cells? [1pt]
Describe the m-skeleton of that CW complex. [1pt]
Describe the n-skeleton of that CW complex. [1pt]

Problem 4 State the definition of homotopy equivalence. [2pt] [1pt]
Prove that if X and Y are two spaces that are homotopy equivalent, then the products $X \times S^1$ and $Y \times S^1$ are also homotopy equivalent. [1pt]

Problem 5 Consider a triangulation of $T^2 \# T^2$ such that at every vertex, exactly seven triangles meet. How many triangles are there in total in that triangulation? [3pt]

Problem 6 The surface $T^2 \# T^2$ admits a CW complex structure whose 1-skeleton is the following graph:

Γ : 

Describe an attaching map $f : S^1 \to \Gamma$ such that $\Gamma \cup_f e_2 = T^2 \# T^2$. [2pt]