**Midterm exam *Topologie en Meetkunde* (WISB341).** A. Henriques, April 2013.
Do not simply provide answers: justify all your assertions.

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**Problem 1** State the definition of a manifold and prove that $S^n := \{x \in \mathbb{R}^{n+1} : \|x\| = 1\}$ is a manifold (disregard the "Hausdorff" and "paracompact" conditions). [3pt]

**Problem 2** Recall the definition of the Euler characteristic $\chi(M)$ of a surface. [3pt] [1pt]
Prove that if $M$ and $N$ are surfaces, then $\chi(M\#N) = \chi(M) + \chi(N) - 2$. [2pt]

**Problem 3** • Recall the definition of $\mathbb{C}P^1$ as a quotient of $\mathbb{C}^2 - \{0\}$. [4pt] [1pt]
• Show that the two maps

$$\{[z:w] \in \mathbb{C}P^1 : |z| \geq |w|\} \to D^2, \qquad [z:w] \mapsto w/z$$

and

$$\{[z:w] \in \mathbb{C}P^1 : |z| \leq |w|\} \to D^2, \qquad [z:w] \mapsto \overline{z/w}$$

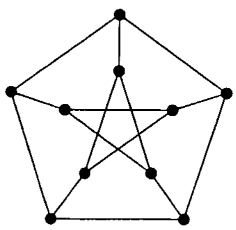
are well defined, and that they are homeomorphisms. Compute their inverses. [1pt]
• Show that the above maps assemble to a homeomorphism from $\mathbb{C}P^1$ to [2pt]

$$S^2 = (D^2 \sqcup D^2)/(x,0) \sim (x,1) \text{ for } x \in \partial D^2.$$

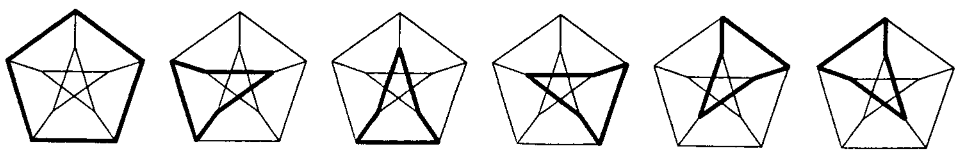
**Problem 4** Let $X$ be a $CW$-complex with one 0-cell and countably infinitely many 1-cells, and let $Y := \{x \in \mathbb{R}^2 : \exists n \in \mathbb{N}, \|x - (1/n, 0)\| = 1/n\}$. We equip $Y$ with the subspace topology from $\mathbb{R}^2$. [3pt]
• Draw a picture of $X$ and of $Y$. [0.5pt]
• Show that there exists a continuous bijection from $X$ to $Y$. [1pt]
• Prove that $X$ and $Y$ are not homeomorphic by arguing that $Y$ is compact, and that $X$ is not. [1.5pt]

**Problem 5** Consider the following graph (called the Petersen graph) [6pt]

$\Gamma$ :



• Prove that $\Gamma$ is homotopy equivalent to $\mathbb{R}^2$ minus six points. [2pt]
• State the classification theorem for compact connected surfaces (without boundary). [2pt]
Let $\Sigma$ be the surface obtained by glueing a 2-cell along each one of the following six 5-cycles of $\Gamma$:



To which surface in the classification is $\Sigma$ homeomorphic? [2pt]