Topologie en meetkunde – Midterm

- **Write your name and student number clearly on on this exam.**
- You can give solutions in English or Dutch.
- You are expected to explain your answers.
- You are allowed to use results of the lectures, the exercises and homework (and you are also allowed to use the results of part (a) and (b) of a problem in part (c) even if you did not solve (a) and (b)).
- Advice: read all questions first, then start solving the ones you already know how to solve or have good idea on the steps to find a solution. After you have finished the ones you found easier, tackle the harder ones.

Problem 1 (15 points). *(a) Give an example of a space X with two points $x, y \in X$ such that $\pi_1(X, x)$ is not isomorphic to $\pi_1(X, y)$.*

(b) Give an example of a surface (i.e. a 2-dimensional compact connected manifold without boundary) that is not homeomorphic to S^2, the torus, the Klein bottle or $\mathbb{RP}^2$ and give its fundamental group. (You can give the result without proof.)

(c) Draw a triangulation of $\mathbb{RP}^2$.

Continuation of Problem 1

Problem 2 (8 points)**.** *Let* $A \subset X$ *be a subspace of a space* X *and* $F\colon I \times X \to X$ *a map such that*

- $F(0, x) = x, \ \forall x \in X,$
- $F(t, x) \in A, \ \forall x \in A, t \in I,$
- $F(1, x) \in A, \ \forall x \in X.$

Show that A *and* X *are homotopy equivalent.*

Problem 3 (15 points)**.** *Consider two disjoint embeddings* $f, g\colon D^2 \to S^1 \times S^1$ *into the torus. Let* X *be* $S^1 \times S^1 \setminus \left(f(\mathring{D}^2) \cup g(\mathring{D}^2) \right)$. *Show that the fundamental group of* X *is isomorphic to* $\mathbb{Z} * \mathbb{Z} * \mathbb{Z}$. *You can choose* f *and* g *as you please for this purpose.*

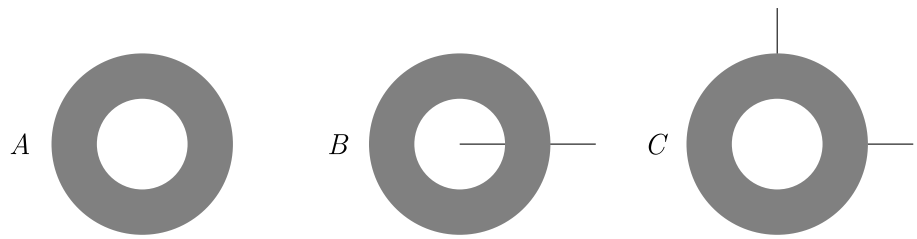
Continuation of Problem 3

Problem 4 (22 points)**.** *Let* $A = \{z \in \mathbb{C} \mid 1 \leq |z| \leq 2\} \subset \mathbb{C}$.

(a) *Let* $f\colon A \to A$ *be a homeomorphism. Show that* $f(z) \in \partial A$ *if* $z \in \partial A$ *and* $f(z) \notin \partial A$ *if* $z \notin \partial A$.

Hint: You are allowed to use the result from the homework that no point in ∂D^2 *has a neighborhood in* D^2 *that is homeomorphic to* $\mathbb{R}^2$.

(b) *Denote for points* $x, y \in \mathbb{C}$ *the line from* x *to* y *by* $L_{x,y}$*. Define* $B = A \cup L_{0,1} \cup L_{2,3}$ *and* $C = A \cup L_{0,1} \cup L_{2i,3i}$*. Let furthermore* X *be as in the previous problem the torus with two disks removed. Which of the spaces* A, B, C *and* X *are homeomorphic and which are homotopy equivalent?*



Continuation of Problem 4