Utrecht University
Mathematics
Stochastic processes
Fall 2011

Test, March 13, 2012

---

**JUSTIFY YOUR ANSWERS**
**Allowed: material handed out in class and *handwritten* notes (*your handwriting*)**

---

**Problem 1.** (10 pts.) Car inspections require, on average, 45 minutes. For Monday morning a garage has scheduled an inspection at 9 AM and another at 10 AM. Both car owners come on time. Assuming that the time required by successive inspections are IID exponentially distributed random variables, determine the expected amount of time the owner of the 10 AM car will spend in the garage.

**Problem 2.** Let $\{N(t) : t \geq 0\}$ be a Poisson process with rate $\lambda$. Let $T_n$ denote the $n$-th inter-arrival time and $S_n$ the time of the $n$-th event. Find

(a) (8 pts.) $E[S_{10}]$.

(b) (8 pts.) $P(T_4 > 3/\lambda \mid T_3 > 2/\lambda)$.

(c) (8 pts.) $E[N(15) \mid N(3) = 4]$.

(d) (8 pts.) $E[S(17) \mid S(16) = 4]$.

**Problem 3.** Let $\{N(t), t \geq 0\}$ be a Poisson process with rate $\lambda$ and $T$ an exponential random variable of rate $\mu$, independent of the process. Consider $s < t$. Find:

(a) (7 pts.) $P(N(T) \geq 2)$.

(b) (7 pts.) $E[N(T^2)]$.

**Problem 4.** Consider a pure death process with three states That is, a process whose only non-zero rates are the death rates $\mu_1$ and $\mu_2$.

(a) (10 pts.) Write the five non-trivial forward Kolmogorov equations.

(b) (7 pts.) Find $P_{11}(t)$ and $P_{22}(t)$.

(c) (7 pts.) Find $P_{10}(t)$.

**Problem 5.** After being repaired, a machine functions for an exponential time with rate $\lambda$ and then fails. The repair process proceeds sequentially through 3 distinct phases. First, a phase 1 must be performed, which is completed in an exponential time with rate $\mu_1$. Then a phase 2 is performed requiring an exponential time with rate $\mu_2$ (independent from the other times). Finally, the time required to complete phase 3 is independent and exponentially distributed with rate $\mu_3$.

(a) (4 pts.) Why is this process *not* a birth-and-death process?

(b) (8 pts.) Determine the limiting probabilities for the process.

(c) (8 pts.) Assume that $\mu_1 = \mu_2 = \mu_3 = \mu$. Determine the ratio $\lambda/\mu$ needed to ensure that, in the long run, the machine is working 40% of the time.