Utrecht University
Introductory mathematics for finance
WISB373
Winter 2014

Exam, May 28, 2014

JUSTIFY YOUR ANSWERS

Allowed: Calculator, material handed out in class, *handwritten* notes (*your handwriting*)

NOT ALLOWED: Books, printed or photocopied material

NOTE:

- The test consists of six exercises.
- The score is computed by adding all the credits up to a maximum of 10 (out of a total of 12 credits, thus there are 2 bonus credits throughout the exam).

Exercise 1. [Mortgage with adjustable interest] To buy a house your subscribe a loan for 200000 euros to be reimbursed monthly during 20 years. The contract determines that the bank has the right to adjust the interest every 5 years according to market values.

(a) (0.5 pts.) At the moment the loan is signed the effective interest rate is 4% per year. Determine the amount of the monthly payment initially agreed.

(b) (1 pt.) The interest is adjusted only after 10 years, at which time it grows to 5% per year. Determine the new monthly payment.

Exercise 2. [Investement abroad] (1 pt) A British bank sells pounds now at a rate of 1.3 E/pound and agrees to buy them one year from now at a rate of 1.25 E/pound. The British bonds pay an interest of r_{Pd} per year. A Dutch investor wants to invest euros now and collect the final payment also in euros. Find the interest r_{E} a continental European bond should pay in the same period, in order not to offer the investor an opportunity of arbitrage through investments on the other side of the Channel.

Exercise 3. (Conditional expectations as martingales) Consider a probability space $(\Omega, \mathcal{F}, P)$ and a filtration $\mathcal{F}_0, \mathcal{F}_1, \cdots, \mathcal{F}_n, \cdots$ Let X be a random variable and consider the random variables

$$M_n \;=\; E(X \mid \mathcal{F}_n)$$

Show that

(a) (1 pt.) the sequence M_n is a martingale adapted to the given filtration, while

(b) (1 pt.) the sequence M_n^2 is a submartingale adapted to the given filtration.

Exercise 4. (1 pt.) Consider two stopping times τ_1 and τ_2 on a probability space and a given filtration. Show that $\tau = \tau_1 \cdot \tau_2$ is also a stopping time for the same filtration.

(Turn over, please)

Exercise 5. (Asian option) Consider a stock with initial price S_0 whose price, at the end of each period, has a probability p of growing 20% and a probability $1-p$ of decreasing 20%. Bank interest is 10% for each period. An investor considers an Asian put option with strike value S_0, that is an option that can only be exercised at the end of the third period, with payoff

$$V_3 = \left| S_0 - \frac{1}{3}\sum_{j=0}^{3} S_j \right|_+ .$$

For the evolution over 3 periods compute:

(a) (0.5 pts.) The risk-neutral probability. [*Hint:* $V_3(HHH) = V_3(HHT) = V_3(HTH) = 0$.]

(b) (1 pt.) The initial price V_0 of the option.

(c) (1 pt.) The hedging strategy Δ_n $(n = 0, 1, 2)$ of the seller.

(d) (1 pt.) The average *net market* payoff as a function of p. That is, the market average of the payoff minus the initial payment translated to the end of the 3rd period. Show that for some values of p it is on the average convenient for the investor to purchase the option, while for some other values it is not

Exercise 6. (Asian American option) In the same setup as in the previous exercise, the investor is offered, as an alternative, the American version of the preceding option. This is an option that can be exercised at the end of any period, and offers intrinsic payoff.

$$G_n = S_0 - \frac{1}{n+1}\sum_{j=0}^{n} S_j \qquad n = 1, 2, 3 .$$

Compute, for this option:

(a) (1 pt.) The initial price V_0.

(b) (1 pt.) The optimal exercise times for the investor.

(c) (1 pt.) The hedging strategy Δ_n $(n = 0, 1, 2)$ of the seller.