Utrecht University
Introductory mathematics for finance
WISB373
Spring 2017

Exam July 17, 2017

JUSTIFY YOUR ANSWERS!!

Please note:

- **Allowed: calculator, course-content material and notes *handwritten by you***
- **NO PHOTOCOPIED MATERIAL IS ALLOWED**
- **NO BOOK OR ADDITIONAL PRINTED MATERIAL IS ALLOWED**
- **If you use a result given as an exercise, you are expected to include (copy) its solution unless otherwise stated**

NOTE: The test consists of five questions for a total of 11 points. The score is computed by adding all the credits up to a maximum of 10

Exercise 1. [Loan with variable interest] To buy a home, a person subscribes a loan for 200000E to be reimbursed monthly for 20 years. The bank keeps the right to change the interest during the reimbursement period.

(a) (0.5 pts.) Determine the monthly payments if the (initial) interest is 6%.

(b) (0.5 pts.) At the end of 10 years the bank reduces the interest to 4%. Find the monthly payment for these last 10 years.

Exercise 2. [True or false] Determine whether each of the following statements is true or false. If true provide a proof, if false provide a counterexample (you can copy examples from class notes or homework problems).

(a) (0.3 pts.) $P(A \cup B) = P(A) + P(B) \Longrightarrow A \cap B = \emptyset$.

(b) (0.3 pts.) $A \cap B = \emptyset \Longrightarrow A$ and B independent.

(c) (0.3 pts.) A and B independent $\Longrightarrow A$ and B^{c} independent.

Exercise 3. [Martingales and submartingales] A biased coin, with a probability p of showing head, is repeatedly tossed. Let $(\mathcal{F}_n)$ be the filtration of the binary model, in which $\mathcal{F}_n$ are the events determined by the first n tosses. A stochastic process (X_j) is defined such that

$$X_j = \begin{cases} 1 & \text{if } j\text{-th toss results in head} \\ -1 & \text{if } j\text{-th toss results in tail} \end{cases} \qquad \text{for } j = 1, 2, \ldots$$

Consider the process

$$\begin{aligned} M_0 &= 1 \\ M_n &= \exp\Big(\sum_{j=1}^{n} X_j\Big) \end{aligned}$$

(a) (0.7 pts.) Determine the values p for which (M_n) is (i) a martingale, (ii) a sub-martingale and (iii) a super-martingale adapted to the filtration $(\mathcal{F}_n)$.

(b) (0.4 pts.) Compute $E(M_n)$.

Exercise 4. [Asian option] Consider the two-period binary market defined by the following values:

		$S_2(HH) = 12$
	$S_1(H) = 8$ $r_1(H) = 10\%$	
		$S_2(HT) = 8$
$S_0 = 4$ $r_0 = 10\%$		
		$S_2(TH) = 8$
	$S_1(T) = 2$ $r_1(T) = 15\%$	
		$S_2(HT) = 2$

(a) An investor is offered an American call option that guarantees buying the stock at the present or immediately preceding price, whichever smaller. That is, at each period $n = 0, 1, 2$ the option has intrinsic values

$$G_n \;=\; S_n - \min\{S_{n-1}, S_n\} \;.$$

-i- (1 pt.) Compute the initial price V_0^{Am} of the option.

-ii- (1 pt.) Establish the optimal exercise time τ^* for the investor.

-iii- (1 pt.) Verify the validity of the formula

$$V_0^{\mathrm{Am}} \;=\; \widetilde{\mathbb{E}}\Big[\mathbb{I}_{\{\tau^* \le N\}} \overline{G}_{\tau^*}\Big] \;.$$

-iv- (0.5 pts.) Show that the discounted values $\overline{V}_n$ do *not* form a martingale.

-v- (0.5 pts.) Determine the consumption process.

-vi- (0.5 pts.) Indicate the hedging strategy for the issuer of the option.

(b) (1 pt.) As an alternative, the investor is offered the European version of the option, namely an option that can only be exercised at the end of the second period and yielding

$$V_2 \;=\; |S_2 - \min\{S_1, S_2\}|_+ \;.$$

Compute the price V_0^{Eu} of this option

(c) (0.5 pts.) Explain why your results do not contradict a theorem, seen in class, stating that some American call options have optimal exercise time at maturity and, hence, cost the same as the American version.

Exercise 5. [American vs European] (1 pt.) Prove that the initial value of an American option is larger or equal than the initial value of its European version.