Natuur- en Sterrenkunde, Universiteit Utrecht

NS-374B: Modern Astrophysics and Observational Cosmology
Mid Semester Test (2012/13)

April 17th 2013

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1. Stellar parallax would be easier to measure if
   (a) the stars were brighter
   (b) the Earth was in the orbit of Venus
   (c) the Earth was in the orbit of Mars
   (d) the Earth moved faster along its orbit
   (e) the measurements were done at the Equator

2. What is the radius and luminosity of a star that is classified as G2 III?
   (a) About 1 solar radii and 10 solar luminosity
   (b) About 5 solar radii and 50 solar luminosity
   (c) About 10 solar radii and 100 solar luminosities
   (d) About 50 solar radii and 1000 solar luminosities
   (e) About 1000 solar radii and 1000000 solar luminosities

3. What effect does the formation of negative hydrogen ions in the Sun's photosphere have on solar observations?
   (a) It cools down the photosphere.
   (b) Concentrations of such ions around sunspots allow us to track solar rotation.
   (c) It reduces the strength of hydrogen lines with respect to Calcium lines
   (d) These ions absorb different wavelength photons which increasing the opacity of the photosphere.
   (e) These ions produce limb darkening because they absorb photons in the visible window of the spectrum.

4. A core-collapse is homogenous when the mass density of
   (a) the inner core remains constant and homogeneous
   (b) the outer core stays homogeneous as the particles infall at a speed $v \propto r$
   (c) the inner core stays homogeneous as the particles infall at a speed $v \propto 1/\sqrt{r}$
   (d) the inner core stays homogeneous as the particles infall at a speed $v \propto r$
   (e) both the inner and outer core remain homogeneous

5. Giant and supergiant stars are rare because

(a) they are not in the main sequence
(b) they are unstable
(c) these stages are short compared with main sequence stage
(d) their luminosity is very large
(e) they lost most of their envelope

6. The stars in a star cluster have approximately the same

(a) age
(b) size
(c) luminosity
(d) spectral type
(e) distance from the Sun

7. If the stars at the turnoff point of a cluster have a mass of $2M_\odot$, what is the age of the cluster?

(a) 5.0×10^6 years
(b) 6.4×10^8 years
(c) 1.4×10^9 years
(d) 3.0×10^{10} years
(e) 1.6×10^{11} years

8. In an asymptotic giant branch star $\kappa \propto \frac{\rho}{T^{3.5}}$. Stellar pulsation occurs because

(a) an increase in density heats up the star causing it to expand like a hot air balloon
(b) an increase in density in a partially ionised region leads to further ionisation of the region. This brings down the temperature causing the star to contract. As it contracts the opacity becomes so high that much pressure is built up in the inner region causing the star to expand and reverse the contraction.
(c) in partially ionised regions the temperature cools down because thermal energy is used to complete the ionisation. The resulting increase of opacity leads to a build up of pressure that eventually causes the star to expand. As the expansion slows down gravitational attraction will reverse expansion and the star starts to contract.
(d) in partially ionised regions an increase in density leads to further ionisation so the temperature heats up only very slightly. The resulting increase of opacity leads to a build up of pressure that eventually causes the star to expand. As the expansion slows down gravitational attraction will reverse expansion and the star starts to contract.
(e) the star pulsates because deep convection swaps H burning with He burning regions. Because the fusion burning temperature of these regions is different when the hotter He burning region is in the outer region the star expands and vice-versa.

9. Core collapse causes a supernova explosion because

   (a) the contraction is suddenly stopped by the onset of neutron degeneracy pressure which sends a shock waves outwards that blasts the stars envelope.

   (b) the core becomes so dense and hot that all its thermal photons materialise into a runaway production of electron-positron pairs that disintegrate the inner core.

   (c) the slower contracting outer core eventually crashes into the inner core which stopped contracting due to neutron degeneracy pressure. The crash releases the neutrinos that have been trapped in the inner core causing the explosion.

   (d) neutrinos trapped in the inner core eventually escape and build up a pressure against the stalled shock wave produced at the end of the collapse. This leads to the explosion.

   (e) neutrinos trapped in the inner core eventually escape and re-ignite an explosive run away fusion in the outer envelope of the star

10. The strongest evidence for a C burning mechanism as the cause of SN type Ia is

    (a) strong O II lines

    (b) strong Si II lines

    (c) the light curve decays with the lifetime of radioactive Ni

    (d) the light curve decays with the lifetime of radioactive Fe

    (e) strong C emission lines in the supernovae remnant spectrum

11. The pressure in a completely degenerate electron gas is $P = \frac{8\pi}{3h^3}\int_0^{p_F} \frac{p^4c^2}{\sqrt{p^2c^2 + m_e^2c^4}}\,dp$ where p_F is the Fermi momentum. In the ultra-relativistic limit and the non-relativistic limit the pressure is respectively given by

    (a) $\frac{8\pi}{3h^3}p_F^3c$ and $\frac{8\pi}{3h^3}\frac{p_F^4}{m_e}$

    (b) $\frac{2\pi}{3h^3}\frac{p_F^4}{m_e}$ and $\frac{8\pi}{9h^3}p_F^3c$

    (c) $\frac{2\pi}{3h^3}p_F^4c$ and $\frac{8\pi}{15h^3}\frac{p_F^5}{m_e}$

    (d) $\frac{8\pi}{3h^3}p_F^4c$ and $\frac{8\pi}{3h^3}\frac{p_F^5}{m_e}$

    (e) $\frac{8\pi}{15h^3}\frac{p_F^5}{m_e}$ and $\frac{2\pi}{3h^3}p_F^4c$

12. (a) During glitches the north and south poles of a pulsar are reversed

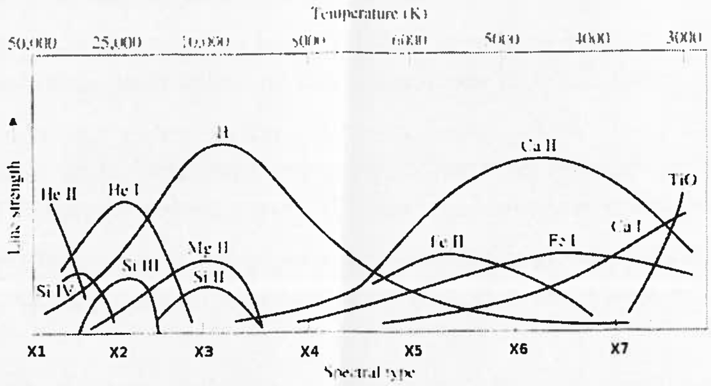
    (b) During glitches the frequency of a pulsar decreases

    (c) During glitches the period of a pulsar decreases

    (d) During glitches the magnetic field of a pulsar decreases

    (e) During glitches the radio emission of a pulsar is not detected

13. Consider the graph showing the line strength for the spectral lines of different elements found in stellar atmospheres.



Answer the following questions by writing **not more than three lines** per question with brief justifications.

(a) Associate a spectral type to each tick Xn, $n = 1, \dots, 7$.

(b) What is the spectral type of a star with strong He I lines?

(c) Why are the absorption lines of Fe II not stronger than the Balmer lines in the Sun atmosphere?

(d) Which is the most prominent element in the spectrum of a brown dwarf?

14. Consider the structure equation of a white dwarf and the equation of state for the ultra relativistic degenerate electron gas, respectively,

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho \quad \text{and} \quad P = K_2\rho^{4/3}, \text{ where } K_2 = 4.921 \times 10^9 \text{ in SI units.}$$

Let R be the radius of this stellar object.

(a) Show that for a ultra relativistic white dwarf the structure equation can be reduced to $\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2\dfrac{dy}{dx}\right) = -y^3$, where $x = r/R$ and $y = (\rho/\rho_0)^{1/3}$, and express ρ_0 in terms of R.

(b) This equation can be integrated numerically with the boundary conditions $y(1) = 0$ and $dy/dx|_{x=0} = 0$. Justify the choice of these conditions.

(c) Numerical integration gives $y(0) \approx 6.9$ and $dy/dx|_{x=1} \approx -2.018$. Evaluate the mass of the white dwarf $M = \displaystyle\int_0^R dr\, 4\pi r^2 \rho(r)$. Discuss your result.

Astronomical and Physical Parameters and Constants

$M_\odot = 1.989 \times 10^{30}$ kg	mass of the sun
$R_\odot = 6.9599 \times 10^{8}$ m	radius of the sun
$L_\odot = 3.826 \times 10^{26}$ J/s	luminosity of the sun
$T_\odot = 5770$ K	surface temperature of the sun
$M_\oplus = 1.989 \times 10^{30}$ kg	mass of the sun
$R_\oplus = 6.9599 \times 10^{8}$ m	radius of the sun
23 h 56 m 0.09054 s	Sideral day
86400 s	Solar day
3.155815×10^{6} s	Sideral year
3.155693×10^{6} s	Tropical year
1 ly = 9.4605×10^{15} m	light-year
1 pc = 3.0857×10^{16} m = 3.2616 ly	parsec
1 AU = 1.496×10^{11} m	Astronomical Unit
$h = 6.63 \times 10^{-34}$ J·s	Planck's constant
$e = 1.60 \times 10^{-19}$ C	elementary charge
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2	gravitational constant
$c = 3.00 \times 10^{8}$ m s^{-1}	speed of light
$k_B = 1.38 \times 10^{-23}$ J/K = 8.617 eV/K	Boltzmann constant
$\sigma_{SB} = 5.67 \times 10^{-8}$ W·m^{-2}·K^{-4}	Stefan-Boltzmann constant
$m_H = 1.673 \times 10^{-27}$ kg	mass of the proton
$m_{He} = 6.643 \times 10^{-27}$ kg	mass of a Helium nucleus
$m_e = 9.11 \times 10^{-31}$ kg	electron mass
$r_N \approx 10^{-15}$ m = 1 fm (Fermi)	approximate size of atomic nucleus
$\sigma_T = 6.65 \times 10^{-29}$ m^2	Thomson cross-section
$\epsilon_0 = 8.85 \times 10^{-12}$ N·m^2/C^2	permittivity of free space

Cosmological Parameters

$H_0 = 70.4^{+1.3}_{-1.4}$ km s^{-1} Mpc^{-1} (often used 70.7)	Hubble constant
$h:\ H_0 = 100\,h$ km s^{-1} Mpc^{-1}	reduced Hubble constant
q_0	deceleration parameter
$\rho_{\rm crit} = 3H_0^2/8\pi^2 G$	critical density
$\Omega = \rho/\rho_{\rm crit}$	density parameter
Ω_B	Baryonic matter density parameter

Useful Equations

- Boltzmann excitation equation

$$\frac{N_i}{N_j} = \frac{g_i}{g_j}\, e^{-\frac{E_i - E_j}{k_B T}}$$

where $E_n = -\dfrac{m_e^{1} k_B^2 Z^2}{2n^2\hbar^2}$ with $n = i, j$

- Saha Equation

$$\frac{N_{j+1}}{N_j} = \frac{2Z_{j+1}}{N_e Z_j}\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} e^{-\frac{\chi_j}{k_B T}} = \frac{A_j}{N_e}\,(k_B T)^{3/2}\, e^{-\frac{\chi_j}{k_B T}}$$

- Expressions for spherical symmetric systems

$$V = \frac{3}{5}\frac{GM^2}{R}$$

$$I = \frac{1}{2}MR^2$$

- Mass-Luminosity relation: $L \propto M^{2.7}$
- Specific intensity I_λ

$$E_\lambda d\lambda = \mathrm{I}_\lambda d\lambda\ dt\ dA\ \cos\theta\ d\Omega$$

Mean specify intensity $\bar{\mathrm{I}}_\lambda$

$$\bar{\mathrm{I}}_\lambda = \langle \mathrm{I}_\lambda \rangle = \frac{1}{4\pi}\int_\Omega \mathrm{I}_\lambda d\Omega$$

specify energy density u_λ

$$u_\lambda d\lambda = \frac{1}{c}\int_\Omega \mathrm{I}_\lambda d\lambda\ d\Omega = \frac{4\pi}{c}\bar{\mathrm{I}}_\lambda d\lambda$$

specify radiative flux F_λ

$$F_\lambda d\lambda = \int_{\Omega_{hem}} \mathrm{I}_\lambda d\lambda \ \cos\theta \ d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \mathrm{I}_\lambda d\lambda \ \cos\theta \ d\Omega$$

radiation pressure P

$$P = \int_0^\infty P_\lambda d\lambda \quad \text{with } P_\lambda = \frac{dp_\lambda/dt}{dA}$$

and for photons with wavelength between λ and $\lambda + d\lambda$

$$P_\lambda d\lambda = \frac{2}{c} \int_{\Omega_{hem}} \mathrm{I}_\lambda d\lambda \cos^2\theta d\Omega$$

- Mean free path $\ell = \dfrac{1}{n\sigma}$
  Decrease in radiation intensity I_λ: $\quad d\mathrm{I}_\lambda = -\kappa_\lambda \rho \, \mathrm{I}_\lambda \, ds$
  Optical depth $d\tau_\lambda = -\kappa_\lambda \rho \, ds$
- Mass fractions

$$\frac{1}{\mu} = \sum_j \frac{(1+z_j)X_j}{m_j/m_H}$$

- Structure equations

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -G\frac{M(r)\rho(r)}{r^2}$$

$$P(r) = \frac{\rho(r)kT(r)}{\mu(r)m_H} + \frac{1}{3}aT(r)^4$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\,\epsilon(r)$$

$$\frac{dT}{dr} = -\frac{3\,\kappa(r)\rho(r)}{64\,\pi\sigma r^2 T^3} L(r)$$

$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu(r)\,m_H\,GM(r)}{k\,r^2}$$

$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = \left(1 - \frac{1}{\gamma}\right) \frac{T(r)}{P(r)}\frac{dP}{dr}$$

$$PV^\gamma = \text{const}$$

- Core collapse

$$v_s > v_{\text{infall}}^{(inner)} \propto r \text{ and } v_s < v_{\text{infall}}^{(outer)} \propto 1/\sqrt{r}$$

- probability density distribution function $f(p)$

$$\text{example: } P = \frac{1}{3}\int (vp)\, 4\pi f(p) p^2 dp$$

- Pressure in a completely degenerate gas

$$P = \frac{8\pi}{3h^3}\int_0^{p_F} v\, p^3 dp = \frac{8\pi}{3h^3}\int_0^{p_F} \frac{p^4 c^2}{\sqrt{p^2c^2 + m_e^2 c^4}} dp$$

with $p_F = \hbar\,(3\pi^2 n_e)^{1/3} = \hbar\,(3\pi^2 Y_e\, \rho/m_H)^{1/3}$

- Condition for degeneracy

$$\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3m_e k}\left(\frac{3\pi^2 Y_e}{m_H}\right)^{2/3}$$

- Lane-Emden equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\,\theta^n$$

- Chandrasekhar Mass

$$M_{\mathrm{Ch}} \approx \frac{\sqrt{6}}{32\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{Y_e}{0.5}\right)^2 \frac{2.018}{m_H^2} \approx 1.457\left(\frac{Y_e}{0.5}\right)^2 M_\odot$$

- Conservative lower bound for the stability of a rotating dense object

$$\rho_{lb} > \frac{3\pi}{GP^2}$$

- Pulsar period increase rate

$$2\pi\frac{\dot{P}}{P^2} I = \frac{2\pi}{3}\frac{B_P^2 R^6 \sin^2\theta}{\mu_0^2 c^3}\frac{8\pi^3}{P^3}$$