Re-exam ***Fluid Mechanics and transport phenomena*****, 4 July 2018, 13:30-16:30**

Answers may be given in English or Dutch. All 3 exercises have equal weight. Subquestions earn marks according to the weight listed. Please hand in your answers to questions 1 to 3 on *separate* sheets.

(1) **Fluid mechanical principles.** Please first answer with YES or NO; then give a one-line motivation.

(a) (1 point) Does the curl of the velocity field vanish in a laminar boundary layer?
(b) (1 pt) Does a particle moving towards a stagnation point get stuck?
(c) (1 pt) Is the dimension of stream function given by $kg\ m^2\ s^{-1}$?
(d) (1 pt) Do inertial forces play a role in the flow through a capillary tube?
(e) (1 pt) Does the Reynolds number tell us when viscous forces are important?
(f) (1 pt) Do three-dimensional vortex monopoles exist?
(g) (1 pt) When a steady two-dimensional inviscid fluid layer flows over a hill, is its free surface above the hill higher than far away from the hill?
(h) (1 pt) Do long surface gravity waves go faster than short surface gravity waves?
(i) (1 pt) Can the flow of an ideal fluid experience drag?
(j) (1 pt) Can vortex rings stay fixed in space?

(2) **Spin-coating**
Spin-coating is a process with which a thin layer - a film - of a certain material is brought onto a fixed circular bottom plate (as the layer of phosphorus on the inside of a television screen). To do so, a viscous fluid (in which the material is dissolved) is brought onto the bottom, which is subsequently rotated with a large angular velocity Ω: the fluid then spreads radially and forms a thin layer.

To analyse the flow in this film we assume that the fluid has Newtonian properties en possesses a kinematic viscosity ν. To describe the flow we use a cylindrical coordinate frame (r, θ, z), as in Figure 1, with velocity components (u, v, w) in radial, azimuthal and vertical directions. After some time (after the bottom plate has reached steady rotation) a thin film of *uniform* thickness $h(t)$ establishes itself. The flow in the film is axisymmetric and quasi-stationary, and the pressure distribution is quasi-hydrostatic. The air does not exert any shear-stresses on the film. Because the film is very thin and the fluid very viscous, the fluid moves azimuthally, together with the bottom plate, with a speed $v = \Omega r$.

The flow in radial direction is driven by centrifugal forces acting on the fluid: this flow can further be considered as a *creeping flow*.
The radial component of the Navier-Stokes equation is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \qquad (1)$$

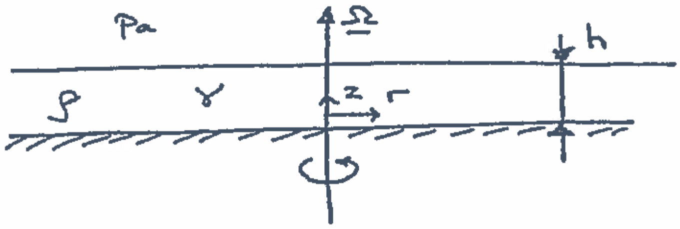

Figure 1: Spinning thin film of fluid

where

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}. \tag{2}$$

(a) (2 pt) Show that under the conditions mentioned and with $h \ll R$ (where R denotes a characteristic radial scale) this equation reduces to:

$$-\frac{v^2}{r} = \nu\frac{\partial^2 u}{\partial z^2} \tag{3}$$

Clearly argue for each neglected term why this can be dropped!
(b) (2 pt) Eq. (3) allows us to determine radial velocity u. Which boundary conditions does u need to satisfy? Derive the solution u.
(c) (2 pt) Axial velocity component, w can subsequently be determined from the continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0. \tag{4}$$

Give the boundary condition that w has to obey at the plate ($z = 0$), and derive the solution for w.
(d) (2 pt) The velocity with which film thickness h changes can be written as:

$$w(z = h) = \frac{dh}{dt}. \tag{5}$$

With the result of (c) this leads to:

$$\frac{dh}{dt} = -\frac{2\Omega^2 h^3}{3\nu}. \tag{6}$$

Determine the general solution for $h(t)$, taking $h(t = 0) = h_0$.
(e) (2 pt) For large times t, $h(t) \ll h_0$, so that the result in (d) reduces to :

$$h(t) = \left(\frac{3\nu}{4\Omega^2 t}\right)^{1/2}. \tag{7}$$

Compute the time needed to acquire a film thickness $h = 0.1$ mm ($= 10^{-4}$ m), when it is given that $\Omega = 10$ cps (cycles per second) and $\nu = 4 \times 10^{-3}$ m^2/s.

(3) Capillary-gravity waves

In a two-dimensional (x, z)-fluid of constant depth H, unbounded in x, capillary-gravity waves, $z = \zeta(x, t)$, propagate along the free surface (of average position $z = 0$). The surface boundary condition that these waves obey reads in good approximation:

$$\frac{\partial \phi}{\partial t}|_{z=0} = -g\zeta + \frac{\gamma}{\rho}\frac{\partial^2 \zeta}{\partial x^2}.$$

Here, ϕ represents a velocity potential, g the acceleration of gravity, and γ surface tension.
(a) (3 pt) List all assumptions that went into the derivation of this boundary condition.
(b) (3 pt) List the additional equations and boundary conditions that govern these waves.
(c) (1 pt) Monochromatic, plane waves $\phi = \Phi \cosh(k(z+H))e^{i(kx-\omega t)}$ of frequency ω, wave number $k = 2\pi/\lambda$ (wave length λ) and amplitude Φ solve these equatioins provided they obey the following dispersion relation:

$$\omega^2 = (gk + \frac{\gamma}{\rho}k^3)\tanh(kH).$$

When can we neglect gravity acceleration? And when capillary effects?
(d) (2 pt) In the deep water limit, $H \gg \lambda$, determine the phase and group velocity of capillary-gravity waves.
(e) (1 pt) For $\gamma = 0.073$ N m^{-1}, $\rho = 1000$ kg m^{-3} and g $= 10$ m^2 s^{-1}, determine the wave length and wave speed for which the wave speed is minimal.