Structure of Matter (NS-266B)

Mid-term exam

5 March 2015. Time: 8:30-10:30 (Please do not leave before 9:15)

The exam consists of two parts:
Exercises on Condensed Matter (Part I) and Subatomic Physics (Part II).

The maximal number of points is indicated for each exercise.
The total number of points is 109.

→ Answer each of the exercises on a separate piece of paper.
→ Write your name and student number on each page.
→ Do not give final answers only, explain your reasoning (short) and give full calculations.
→ Simple calculator use is allowed (no programmable)
→ No mobile/smart phone!

Good luck!

Part I – Condensed Matter

Exercise 1.1: Structure 1 (15 points)

Researchers at Utrecht University have developed the synthesis of cubes with a cube length of 1.0 μm and an interparticle interaction that is to good approximation described by 'hard' interaction potential. They made this into a 2D model system by adsorbed these particles to the air water interface layer where the cubes adsorb with their planes parallel to the interface essentially forming a 2D system of squares. By microscopy the radial distribution function is measured experimentally by microscopy as function of density. It is found that at an area fraction of 0.92 a 2D crystal with an area density of 0.94 starts to form through a first order (meaning there is a density jump) phase transition. In the following provide sketches of the 2D $g(r)$ (pair distribution) functions, including on the x-axis (roughly) were features are to be expected and on the y-axis mark where the unit 1 can be found. Make sketches for the following cases:

a) (5 points) Dilute limit of density where the cubes approximate a 2D ideal gas.

b) (5 points) Area fraction just below the start of the freezing transition.

c) (5 points) Area fraction of 0.95.

Exercise 1.2: Structure 2 (20 points)

The radius of a chloride ion: Cl^- is 0.188 nm, that of a cesium, Cs^+ ion is 0.167 nm and that of a sodium, Na^+ ion is 0.098 nm. In ionic crystals the cation/anion (negative) size ratio: r_+/r_- is an important parameter determining the packing symmetry of the lattice formed.

a) (5 points) Give a schematic drawing of the CsCl lattice structure (and indicate lattice parameter sizes). What is number of direct neighbors of the chloride ion?

b) (5 points) Derive the size ratio r_+/r_- range for which you expect *AB* ionic crystals to adopt the CsCl structure, explain your reasoning.

c) (5 points) Does the size ratio of NaCl fall into the range calculated? Give a schematic drawing of its structure and indicate the number and near neighbors in it.

d) (5 points) What is the size ratio for the NaCl structure to form stable *AB* salts, explain your reasoning and calculations?

Exercise 1.3: Bonding (10 points)

The element carbon is known in many different forms because of its valency. Graphite, graphene and diamond are three examples.

a) (5 points) Describe the structures and different binding forces between the carbon atoms in crystals of these materials.

b) (5 points) Explain why on the one hand graphite is a soft solid, easily destroyed, while both graphene and diamond are known to be extremely strong solids.

Exercise 1.4: Scattering (20 points)

Oppositely equally charged micron-sized (all particles have a size of 1 μm) colloids made from both silica (where both the positive and negative particles are made from silica), and mixtures of silica (negative) and titania (positive) are found to crystallize in a CsCl type colloidal crystal. Comparing the (single) light scattering of these colloidal crystals it is found that one type of crystals misses a set of Bragg peaks visible in the other crystals (a HeNe laser is used with a wavelength of 632 nm):

a) (15 points) Which crystals are missing what kind of Bragg peaks; explain your answer by a derivation of the total sets of Bragg peaks for both these two crystals. (*Only,* if you do not know the 3D structure of CsCl assume it is a two atom basis FCC lattice with one atom at the origin and one at $(0.5,0.5,0.5)a$ if a is the unit cell length).

b) (5 points) At what scattering angle does one find the first Bragg peak?

Part II – Subatomic Physics

Exercise 2.1: Multiple choice questions (20 points)

Instructions: Choose one answer. Each correct answer gives 2 point.

1. The mass of an elementary particle is typically given in non-natural units, namely

   A. MeV
   B. MeV/c
   C. MeV•c^2
   D. MeV/c^2

2. What is the evidence that an atom has a nucleus where about 99% of the mass and the positive charges are concentrated?

   A. A particle scattered from the atom would have a scattering angle much smaller than 1°.
   B. A particle scattered from the atom would have a scattering angle much larger than 1°.
   C. That the neutron consist of a proton and electron.
   D. There is no such proof, and physics still have to investigate it.

3. What is the reason that a neutron does not decay when it is bounded in a nucleus?

   A. Neutrons are shielded from the vacuum and therefore do not decay.
   B. Neutrons interact with other neutrons and protons inside the nucleus by pion exchange.
   C. The neutron is heavier than the proton.
   D. Neutrons decay but their lifetime is extremely high.

4. The radius of an atom is how many times bigger than the one of a nucleus?

   A. 10^2
   B. 10^3
   C. 10^5
   D. 10^7

5. What is the indication in the β-decay of a neutron that a neutral, almost mass-less particle exists (later called neutrino)?

   A. Unexpected fast decay of certain nuclei species.
   B. Continuous energy spectrum of the decay electron.
   C. Discrete energy spectrum of the decay electron
   D. Excitation spectrum of β-particle emitting nuclei.

6. Which statement is correct? Protons and neutrons are

   A. not fundamental particles, evident from their gyromagnetic factor.
   B. not fundamental particles, evident from their different mass.
   C. fundamental particles since they built up nuclei.
   D. fundamental particles since they can be emitted by nuclei.

7. The basic idea of the Bethe-Weizsäcker mass formula is the fact that the nucleus can be considered as an

   A. extended object with a mass density decreasing with the radius.
   B. extended object with a mass density increasing with the radius.
   C. ideal gas.
   D. incompressible liquid.

8. The nuclear shell model gives a good description of the

   A. radius and mass of nuclei.
   B. collective motion of valence nucleons inside the nuclei (e.g., vibration and rotation)
   C. structure and excitation spectra of nuclei.
   D. potential of nuclei.

9. The nuclear force is

   A. a fundamental force.
   B. strongly spin dependent.
   C. a long-range force.
   D. weaker than the electromagnetic force.

10. Feynman diagrams

   A. provide computational rules to describe particle interactions.
   B. are a graphical summary to describe the nuclear force.
   C. can only be used for the description for interaction processes in nuclei.
   D. describe only the interaction between protons and neutrons inside nuclei.

Exercise 2.2: Age determination with ^{235}U:^{238}U (7 points)

According to the current understanding of the nucleo-synthesis, the two Uranium nuclides ^{235}U and ^{238}U are formed by rapid neutron capture (the so-called *r-process*) in a supernova event. There is no reason to create one more frequently than the other. Therefore, the today's very different abundance is related to the different decay times.

Determine the time that has passed after the element synthesis by calculating the ratio $R(t) = N_{235}(t) : N_{238}(t)$. Assumption: $R(0) = 1$.

The abundance and half life-time are listed in the table. For the time constant applies $\tau = t_{1/2} / \ln 2$.

Nuclide	Abundance [%]	Half life-time [years]
^{235}U	0.72	$7.1 \cdot 10^8$
^{238}U	99.275	$4.5 \cdot 10^9$

Exercise 2.3: Semi-empirical mass formula (7 points)

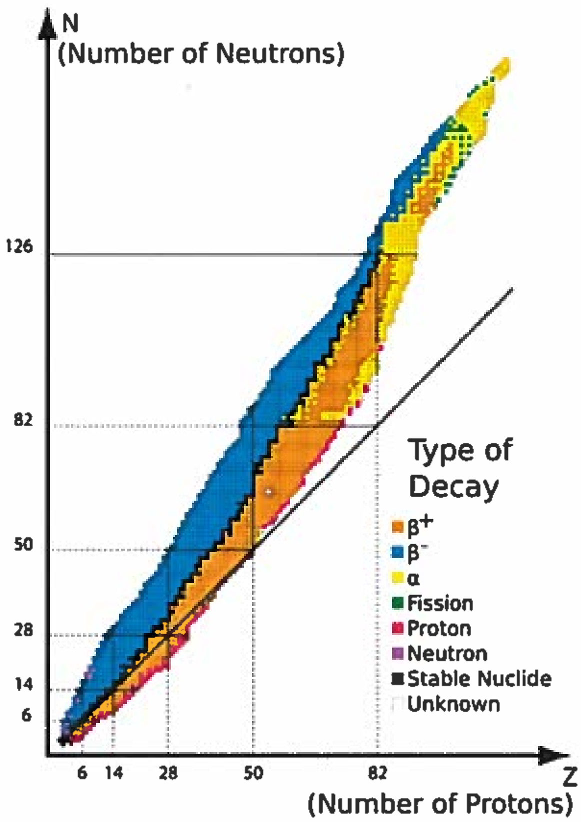
Consider a nucleus with $A = 237$. Use the semi-empirical mass formula

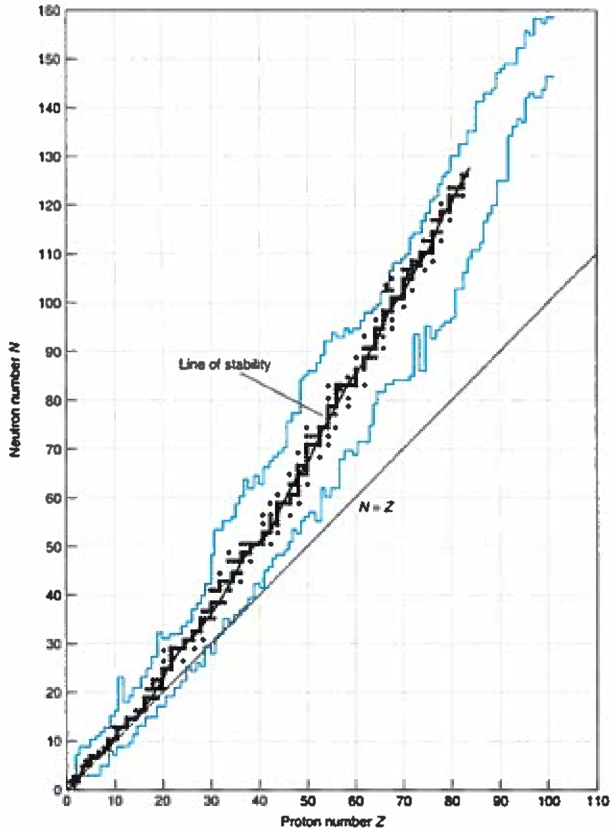
$$E_B = a_v A - a_s A^{2/3} - a_{symm}\frac{(2Z-A)^2}{A} - a_c\frac{Z^2}{A^{1/3}},$$

where $a_v = 15.6$ MeV, $a_s = 16.8$ MeV, $a_{symm} = 22.5$ MeV and $a_c = 0.72$ MeV

to:

a) (5 points) Find Z for the most stable isobar.
b) (2 points) Discuss the stability of this nuclide for various likely decay modes using the nuclide charts below.





Same data on the grid.

Exercise 2.4: Decay of heavy nuclei (10 points)

Consider the nucleus decay A → B + X. Particle A is initially at rest. Calculate the momenta (p_B and p_X) as well as the kinetic energy (E_B and E_X) from the released energy of the reaction (= Q value). Calculate p and E for the following decays:

a) (5 points) ^{238}U → ^{234}Th + α, $\quad Q = 4.268$ MeV $\quad t_{1/2} = 4.5\cdot10^9$ years

b) (5 points) ^{57}Fe ($E = 14.4$ keV) → ^{57}Fe (ground state) + γ $\quad \tau = 10^{-7}$ s

Useful conversion factor: 1 u = 931 MeV/c^2 = $1.66054\cdot10^{-27}$ kg