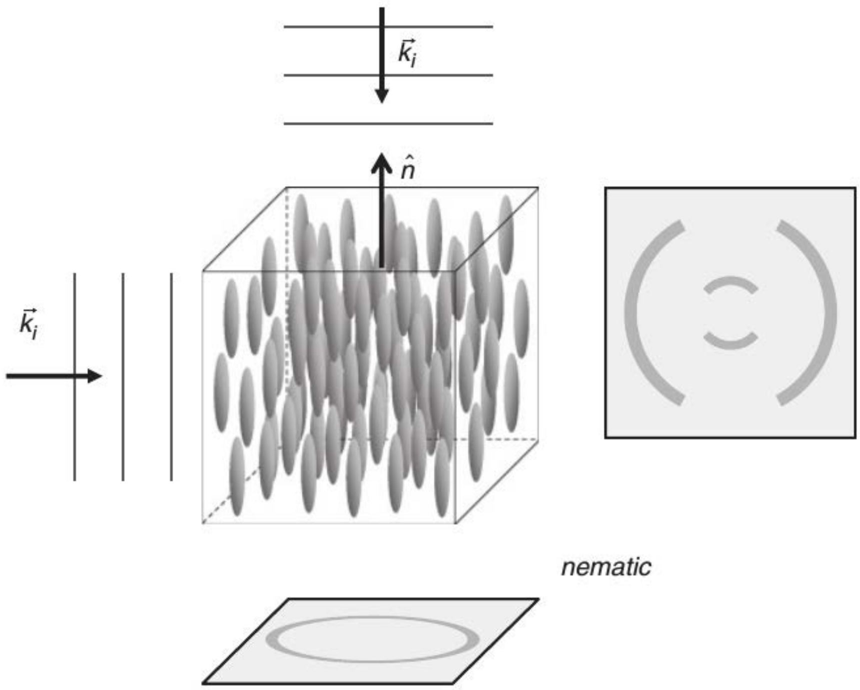
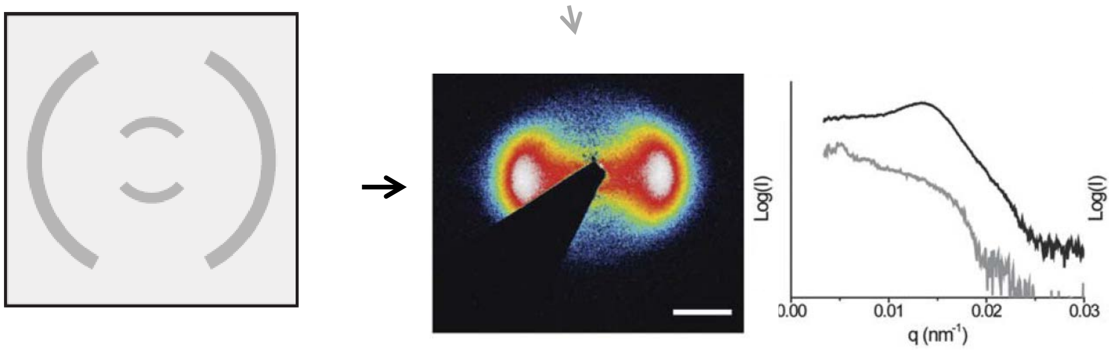
Structure of Matter (NS-266B)

Exam – solutions

Solution 1.1

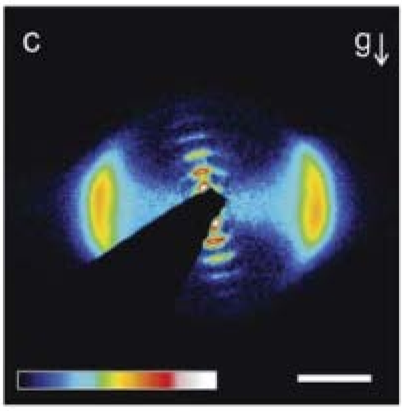
a) From the 2D scattering pattern we identify the Nematic liquid crystal phase, where elongated particles have long range orientational order – they all align in one common direction, called the nematic director – but only short range positional order – meaning the positions of the particles' center of mass is random, fluid-like.
The scattering pattern is taken when the incident beam has wavevector orthogonal to the nematic director, hence the experimental setup looks like:

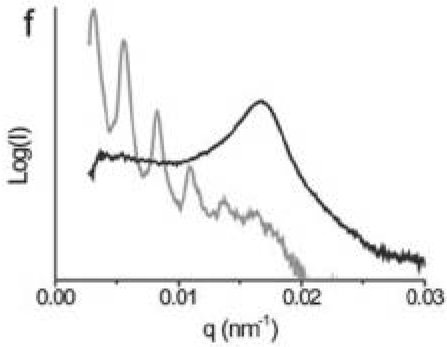




So, the comparison is to be made with these the following patterns

If we now look at the intensities along the different lines, we see a clear peak at q=0.015 (black line) and a very peak at q=0.005 (grey line). Because scattering vectors are related to distances by inverse proportionality, it is clear that the peak in the black line measures the smallest size in the system (interparticle spacing), while the peak in the grey line measures the largest size in the system (particle length). This will be used in point b.

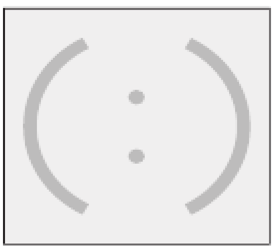




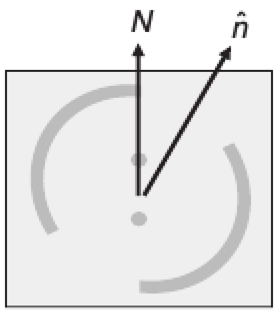
REMARK: Notice that, ONLY because of unclear pictures in Sidebottom's book, we gave full points even if the answer was SMECTIC phase. In this phase, the orientation of the particles is also fixed, but now the particle position have long range order in one direction (say z), while still disordered in the other two directions (say x,y). This results in the system forming consecutive 'layers' of commonly oriented particles, which should generate many bragg peaks along the 'grey' direction (not just two). This is, in fact, the case if we take an experimental picture

where we see many orders of bragg peaks due to the layers in the z direction. Correspondingly, also the grey line in the intensity shows many peaks.

Now, because Sidebottom's book only displays two of such bragg peaks, we thought the students could easily answer also 'smectic phase' (which happened). We gave thus full mark also for the smectic A/C answer, because of the experimental picture and the book not being 100% clear on this.



smectic (A)



smectic (C)

b) If we now look at the intensities along the different lines, we see a clear peak at q=0.015 (black line) and a very peak at q=0.005 (grey line). Because scattering vectors

are related to distances by inverse proportionality, it is clear that the peak in the black line measures the smallest size in the system (interparticle spacing), while the peak in the grey line measures the largest size in the system (particle length).
The calculation is then:
- a (interparticle distance) = 2*pi/q_peak_black = (6.283/0.015) nm ~ 420 nm
- l (particle length) = 2*pi/q_peak_grey = (6.283/0.005) nm ~ 1250 nm

c) This question is best answered starting from the functioning principles of an LCD cell. This cell is made out of two crosspolarised filter at a certain distance and it is filled with a **molecular** liquid where the particles are elongated in one direction. The inner walls of the polarisers are coated in such a way that they encourage the molecules in the fluid to align their long side along the polarisation direction, say along x for the top layer and along y for the bottom layer. Because of this wall effect, the nematic director twists continuously from the x direction to y direction as one goes from the top of the cell to the bottom. This twist in the nematic director induces a twist in the x-polarised light that goes through the top filter, resulting in a y-polarised light at the bottom of the cell. This light now agrees with the bottom filter polarisation and can go through it, so the cell is in the 'on' state by default (without applying any work to it). If we now apply a voltage to the two polarisers, the electric field inside the cell will promote particles to align in the z direction and the cell cannot twist the polarisation of the light anymore. This polarisation if perpendicular to the bottom filter, so no light comes out of the cell when the electric field is on and the cell is in an 'off' state. Note that, since the external control knob is 'on' means the cell is 'off' and viceversa, this is an example of what is called 'inverse logic', as opposed to the 'direct logic' (knob 'on' → cell 'on').

From this discussion, it is clear that a system composed of elongated particles is the main ingredient of the LCD, as long as the wall coating ensures a continuous twist of the nematic director. Thus, this concept could also be applied to the liquid crystal at hand, our nematic phase. However, there are some limitations:
1) Size matters! Molecular liquids have particles of much smaller dimension (particle size ~ Angstroms) and respond much faster to external stimuli, whereas a colloidal liquid crystal (particle size ~ um) responds much slower, making the whole switch particularly 'slow'
2) The scattering from a colloidal LC is much more than a molecular one, this effect can be minimised but not eliminated.

Grading 1.1

a)
+1 pts for correctly identifying a nematic or a smectic phase
+2 pts for discussing the short-range positional order and the long-range orientational order
+2 pts for the correct drawing of a nematic or a smectic phase
+3 pts for indicating correctly from which direction, with respect to the nematic director, the beam shines
-2 pts for incorrectly relating scattering units and lengths units
-1 pts for writing down incorrect explanations

b)
+2 pts for discussing how the black line, clear peak is related to length
+1 pts for correctly calculating the interparticle spacing
+2 pts for doing the same, but for the peak at smaller q from the grey line, yielding the particle size
-1 pts if no discussions on the intensity peaks is found
-1 pts for writing down incorrect explanations

c)
+3 pts if correcly explained the functioning principles of an LCD displays
+2 pts for realising that any material capable of forming a LC is suitable for the LCD display (hence also the given, colloidal LC) and for discussing the limitations of using a collidal LC versus a molecular LC.
-1 pts for writing down incorrect explanations

CM Exercise **1.2** (10 points)
a)

1.2 a) Energy levels similar to that of particle-in-a-box:

$E_n = \frac{n^2 \hbar^2 \pi^2}{2 m L^2}$ / $E_K = \frac{\hbar^2 |K|^2}{2m}$ (2)

OR $\Delta K = \pi / L$ (1)

Energy level spacing at the Fermi level for a cubic dot of size 5 nm:

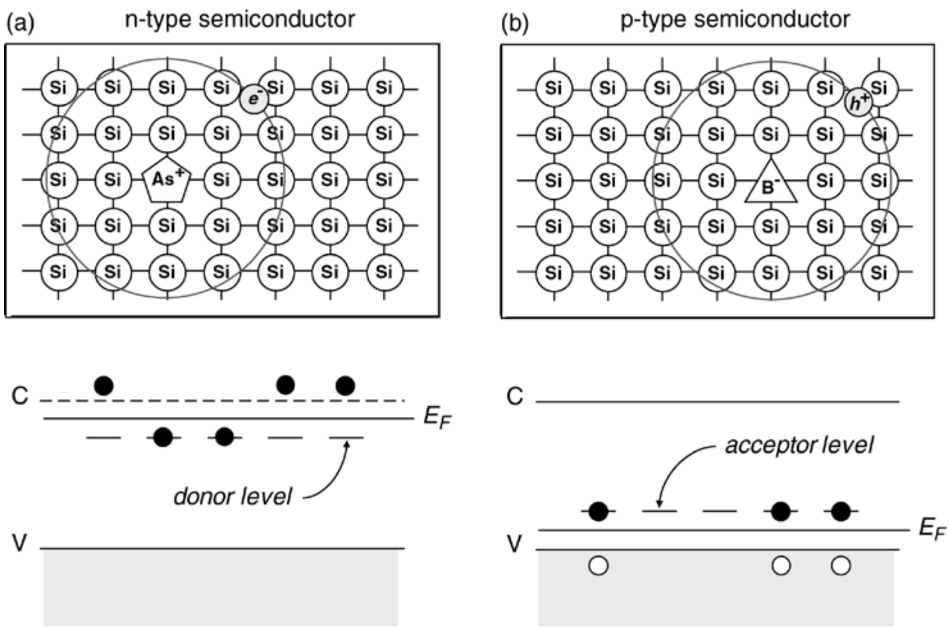
$\Delta E = \left(1/3\pi^2 N_e\right)^{1/3} \frac{\hbar^2 \pi^2}{m L^2}$ → characteristic energy scale. (2)

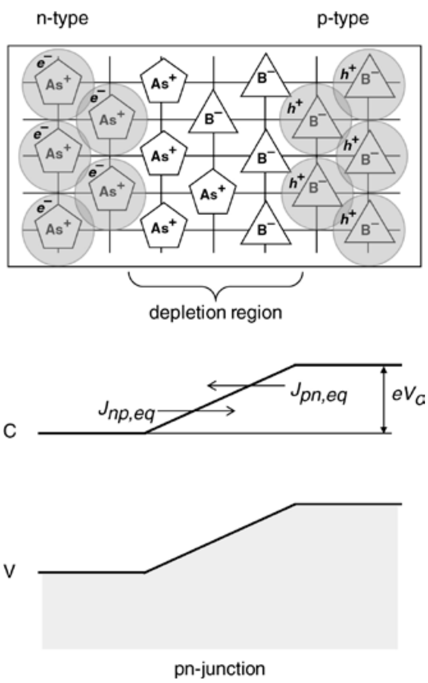
~ 0.01 eV ~ 10 meV

(spacing large enough for length scales of 5 nm) (1)

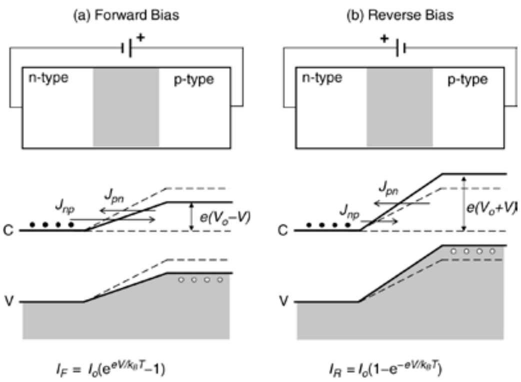
b) An *STM* scanning tunneling microscope moves a very fine conducting needle over a surface and (for instance) makes use of the quantum mechanical phenomenon of tunneling. For a conducting substance there is an exponential relationship between the chance of an electron tunneling from the surface to the needle which is kept at a potential difference with the surface. Piezo electricity is used to keep for instance the current constant while scanning over a surface thus creating a sensitive height map. It is also possible to vary the potential difference and thus perform a kind of spectroscopy with an STM that can say something about the energy levels the electrons are tunneling from (3 points). However, all this is only possible for conductive surfaces (this holds for both a bulk and quantum dot semiconductor). This would be for instance possible with an intrinsic semiconductor with a relatively small band gap and/or doped semiconductors. The modes referred to in a) have very small energy difference which cannot be picked up in the case of a bulk semiconductor. However, both the (much) larger energy differences (and spatial extend) of the wave functions solutions of the particle in a Quantum Dot box could be (and have been) measured by an STM (2 points).

CM Exercise **1.3** (23 points)



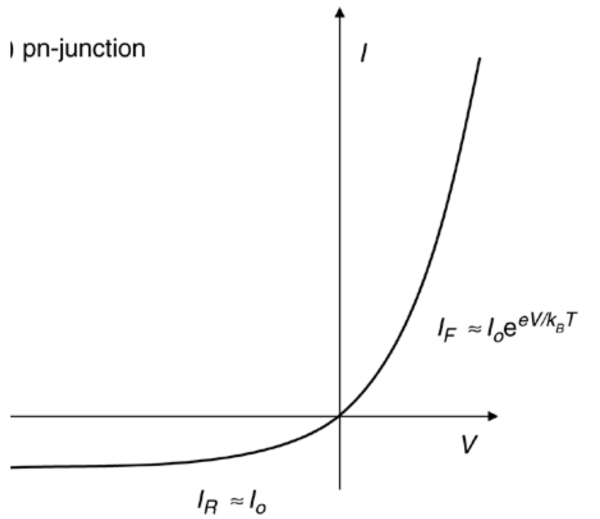


$$J_{np} \propto e^{-eV_o/kT} \approx J_{pn}$$



a) A *pn* junction is created by placing an *n*-type doped silicon single crystal next to a *p*-type silicon single crystal in the sense that one single crystal is formed with a region doped with elements that have an excess of electrons with respect to the number of binding electrons of silicon (such as Arsenic As 2 points) creating an excess electrons in the conduction band (as at room temperature the excess electrons are only weakly bound to the doping atoms and also positioned just below the conduction band (see the diagrams below 2 points). Similarly the silicon crystal can also be doped (e.g. by high energy ion implantation or atomic diffusion induced by temperature: 2 points) with an element which is placed in the periodic table more to the right than the column silicon is in (e.g. boron atoms B: 2 points) resulting in atoms that want to take up an electron from the valence band electrons thus creating an excess of positively charged holes. When brought into contact the excess of electrons from the *n*-doped region combines with the excess of holes in the *p*-doped region creating a depleted region and regions where there is excess negative and positive charge resulting in a potential barrier over the *pn* region this potential serves to raise the energy band diagram on the *p*-type side relative to that on the *n*-type side (diagram + explanation 4 points).

b) (See also a)) The potential barrier created by the chemical potential differences for electrons that are the result of the doping of the silicon single crystals with different amount of trace species brakes the symmetry for electrons flowing in an electric circuit in which a *pn* junction has been placed. This potential barrier creates a Boltzmann exponential barrier in one direction of the current while in the other direction and exponential growth of the current (3 points, see the graph and approximate formula's 2 points) thus creating a circuit which allows current to run one way and opposes it in the other direction (*n* to *p* is limited): rectification: a diode.



c) The unusual temperature dependence of the conductivity is that with increasing temperature the conductivity increases exponentially with temperature, while for metal conductors the resistivity increases (proportionally) with temperature because of the increased resistance caused by phonons (3 points). The derivation of this behavior can be performed using the Fermi-Dirac distribution and the density of states which together (2 points) explain this behavior (see Sidebottom 234): the gap difference should be not too large with respect to *kT* for this to happen at room temperature (1 point). With an *n*-type semiconductor (see a)) there is an excess of electrons in the conduction band from the donor atoms. However, excitation from the valence band to the conduction band still remains possible and thus an increase of conductivity with temperature, except that the effective increase will be (much) less because of the excess of the electrons already present (2 points).

Exercise 2.1: Multiple choice questions

1C
2A
3D
4B
5A

6D
7C
8C
9D
10A

Structuur van de materie tentamen

2.2 a) A) not allowed because both baryon number and electron lepton number are not conserved

B) allowed, because it does not violate any conservation law

C) allowed, only S is not conserved but this is fine in the weak interaction

D) not allowed, $Q = m_p - m_{\Lambda^0} - m_e = -178\,\text{MeV} < 0$. Note that this only has to be checked for <u>decays</u>

E) allowed, because it does not violate any conservation law

F) allowed, because it does not violate any conservation law

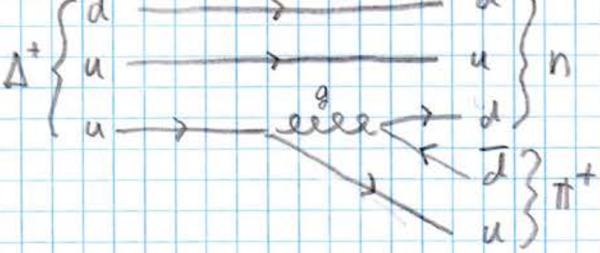
Note: $L_\mu(\mu^+) = L_\mu(\bar{\nu}_\mu)$ and $L_e(e^+) = -L_e(\nu_e)$

<u>points:</u>

- 2 if you correctly say whether the process is allowed or not
- 1 if you incorrectly say whether the process is allowed or not, but explained for each conservation law seperately if it is conserved or not, and only made a single mistake which led you to the wrong conclusion
- 0 if you draw the incorrect conclusion without further checking each law, or if you make multiple mistakes in checking the laws.

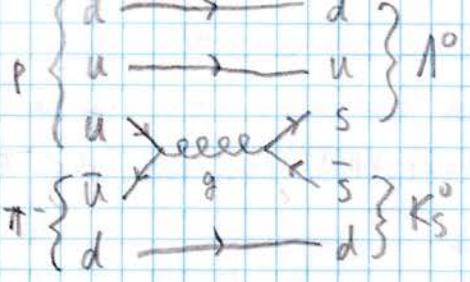
b) A) $\Delta^+(uud) \rightarrow n(udd) + \pi^+(u\bar{d})$

this is a strong process



· B) $\pi^-(\bar{u}d) + p(uud) \longrightarrow \Lambda^0(uds) + K_s^0(d\bar{s})$

This is a strong process (see Tipler page 602)



points

- +1 if you correctly write down the quark content
- +3 if you correctly draw the Feynman diagram on the quark level
- −1 if you use the weak force or EM force
- −1 if you have wrong vertices (e.g. flavor change when emitting gluon)
- −1 if you don't draw arrows

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2.3 a) $p + p \rightarrow p + p + \pi^0$ $m_p = 938\,MeV/c^2$ $m_\pi = 135\,MeV/c^2$

The threshold energy is when in the center of mass frame the available energy is just enough to produce a π^0:

$E_{com} = 2m_p + m_\pi$ $(c=1)$ (1)

To connect the available energy in the com-frame with the beam energy E_{beam} in the lab frame, write down the invariant mass of the combined system of protons in both frames:

$m_{inv}^2 = E_{com}^2$ (center of mass frame)

$= E^2 - |\vec{p}|^2$ (lab frame)

$= (E_{beam} + m_p)^2 - p_{beam}^2 = E_{beam}^2 + m_p^2 + 2E_{beam}m_p - E_{beam}^2 + m_p^2$

where in the last step we have used $p_{beam}^2 = E_{beam}^2 - m_p^2$

This gives us a relation between E_{beam} and E_{com}:

$E_{com}^2 = 2E_{beam}m_p + 2m_p^2$

Filling in the threshold condition (1):

$(2m_p + m_\pi)^2 = 2E_{beam}m_p + 2m_p^2$

$4m_p^2 + m_\pi^2 + 4m_p m_\pi = 2E_{beam}m_p + 2m_p^2$

$\rightarrow E_{beam} = m_p + m_\pi\left(2 + \frac{m_\pi}{2m_p}\right)$

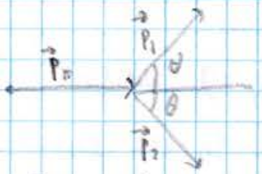
So the proton beam requires a kinetic energy of $m_\pi\left(2 + \frac{m_\pi}{2m_p}\right)$

Filling in the masses results in a threshold kinetic energy of <u>280MeV</u>

<u>points</u>

- +2 if you note that the threshold requirement is $E_{com} = 2m_p + m_\pi$ where E_{com} is the energy available <u>in the center of mass frame</u>
- +2 if you manage to relate lab frame beam energy E_{beam} to E_{com} using the invariant mass $(E_{com}^2 = 2E_{beam}m_p + 2m_p^2)$
- +1 if you obtain the expression for the threshold energy $m_\pi\left(2 + \frac{m_\pi}{2m_p}\right)$
- +1 if you calculate a threshold kinetic energy of 280 MeV (or if you calculated total energy $(280+938)\,MeV = 1218\,MeV$)

b) $\pi^\circ \rightarrow \gamma + \gamma$



$E_\pi = 850$ MeV

$m_\pi = 135$ MeV/c^2

The momentum of the pion is $p_\pi = \sqrt{E_\pi^2 - m_\pi^2} = 839$ MeV/c

Define the x-axis in the direction of the pion momentum and the y-axis perpendicular to that. Then:

$\vec{p}_\pi = (p_\pi, 0)$

$\vec{p}_1 = (p_1 \cos\theta, p_1 \sin\theta)$ momentum of photon 1

$\vec{p}_2 = (p_2 \cos\theta, -p_2 \sin\theta)$ momentum of photon 2

Momentum conservation (in the x-direction) gives us

$p_\pi = (p_1 + p_2)\cos\theta$

Energy conservation gives us

$E_\pi = |\vec{p}_1| + |\vec{p}_2|$ (Since for a photon $E = |\vec{p}|$)

$= p_1 + p_2$

Combining these two gives us $p_\pi = E_\pi \cos\theta$ or

$\cos\theta = \frac{p_\pi}{E_\pi} \rightarrow \theta = \arccos\left(\frac{p_\pi}{E_\pi}\right) = \arccos\left(\frac{839}{850}\right) \simeq 9.14^\circ$

<u>points</u>

- +1 if you calculate $p_\pi = 839$ MeV/c
- +1 if you note that $\theta_1 = \theta_2 \equiv \theta$
- +1 if you use conservation of momentum to obtain $p_\pi = (p_1 + p_2)\cos\theta$
- +1 if you use conservation of energy to obtain $E_\pi = p_1 + p_2$
- +1 if you correctly find the expression $\cos\theta = \frac{p_\pi}{E_\pi}$
- +1 if you calculate $\theta \simeq 9.14^\circ$