Structure of Matter (NS-266B)

Resit

30 June 2015. Time: 13:30-16:30 (Please do not leave before 14:15)

The exam consists of two parts:
Exercises on Condensed Matter (Part I) and Subatomic Physics (Part II).

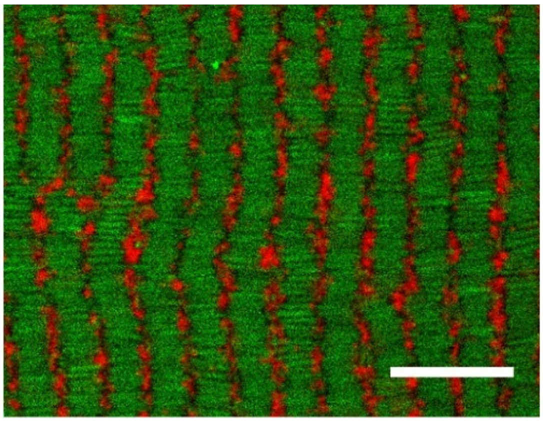
The maximal number of points is indicated for each exercise.
The total number of points is 114.

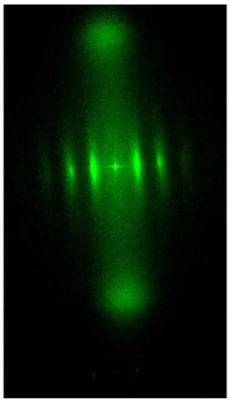
→ Answer each of the exercises on a separate piece of paper.
→ Write your name and student number on each page.
→ Do not give final answers only, explain your reasoning (short) and give full calculations.
→ Simple calculator use is allowed (no programmable)
→ No mobile/smart phone!

Good luck!

Part I – Condensed Matter

Exercise 1.1 (22 points)





1.1) When colloidal 'hard' rods (with cylindrical shape: length 2 μm, diameter 200 nm) are mixed in a solvent with colloidal 'hard' spheres (diameter 200 nm) a so-called binary liquid crystal phase is formed. Binary refers here to the fact that the liquid crystal phase consists of two components (rods/spheres). A microscopy image of this phase is shown in the image on the left (bar is 6 μm, rods and spheres: green and red,). This image only shows particles in one plane (*x, z* plane), the layers of rods and spheres extend perpendicular to the plane (*x, y*) and it is found that both the rods and spheres perform diffusion in these layers. By refractive index matching of the spheres (which makes it that they do not scatter any light) the scattering pattern of the rods of this binary liquid crystal phase is obtained and shown in the image on the right (the light beam used for this scattering experiment was perpendicular to the plane shown in the right image).

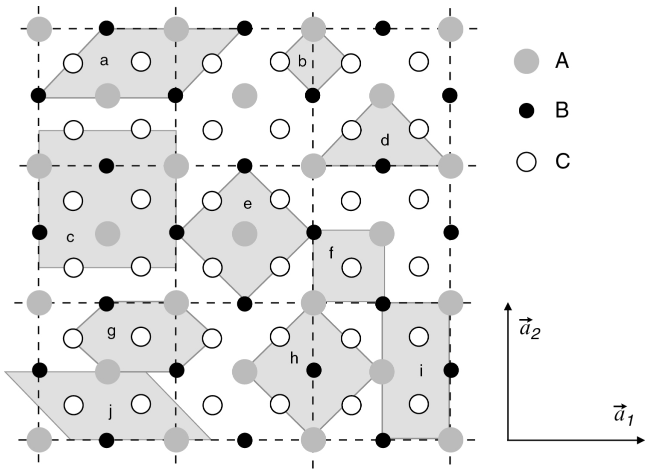
a) (4 points) Which binary liquid crystal phase has formed here? Use in your answer definition of liquid crystal phases from one anisotropic component.

b) (6 points) At what numerical values of the scattering wave vector are the peaks that are visible in the scattering pattern of the rods on the right found? Illustrate the associated distances with a (real space) drawing.

c) (6 points) If instead of the spheres the rods would be refractive index matched, and thus the scattering pattern would be determined only by that of the spheres, how would the 2D scattering pattern look like? Not only illustrate this with a drawing, but also give numerical values of where you expect peaks to appear.

d) (6 points) Give a schematic plot of the pair distribution function of the 2D structure you would expect of the rods in one of the (identical) *x, y* planes perpendicular to the imaging plane of the left figure if a plane is chosen that only contains rods. Explain at what distances the features in your plot are found.

Exercise 1.2 (13 points)



The 2D crystal shown in the Figure above contains three atoms (A, B, C) with a chemical formula ABC_2. Illustrated by grey shading in the figure are several possible tiles (*a-j*).

a) (5 points) Identify which of the tiles are primitive cells and explain why.

b) (3 points) Identify which of the tiles are conventional cells and explain why.

c) (5 points) For each primitive cell, provide expressions for the appropriate basis vectors describing the basis set of atoms.

Exercise 1.3 (25 points)

Silicon (Si) is a so-called *intrinsic* semiconductor with an energy between the conduction and valence band of ~1 eV.

a) (5 points) For many semiconductor applications *intrinsic* semiconductors are turned into so-called *extrinsic* semiconductors. Explain why this is done by using the difference between an *intrinsic and extrinsic* semiconductor.

b) (5 points) Which types of *extrinsic* semiconductors are there, give two examples of applications (in electronics) in which these are used?

c) (5 points) What is the difference between the temperature dependence of the resistivity of metals, like copper, as compared to that of a semiconductor like Si?

d) (10 points) The Fermi-Dirac distribution is given by the following equation:

$$P_{FD}(E) = \{\exp[(E-\mu)/k_B T]+1\}^{-1}$$

Explain what the symbols in the equation mean and use it to derive an expression for the number of electrons in the conduction band of Si, mention the assumptions made in deriving this equation and illustrate why the derived equation explains the unusual temperature dependence mentioned in c).

Part II – Subatomic Physics

Exercise 2.1: Multiple choice questions (20 points)

Instructions: Choose <u>one answer</u>. Each correct answer gives 2 point.

1. The momentum of an elementary particle is typically given in not-natural units, namely

   A. GeV
   B. GeV/c
   C. GeV•c
   D. GeV/c^2

2. The sub-structure of the proton and neutron is evident from its

   A. mass.
   B. electric charge.
   C. colour charge.
   D. magnetic moment.

3. The Rutherford's scattering law holds for α-particles that

   A. are absorbed by the gold nucleus.
   B. penetrate the nucleus.
   C. undergo a deep inelastic scattering.
   D. have a head-on collision on the gold nucleus.

4. At current energies in the universe the nuclear force is

   A. residual of the electro-magnetic force.
   B. residual of the strong force.
   C. is a fundamental force.
   D. as strong as the electro-magnetic force.

5. The nuclear interaction between two nucleons can be considered as the exchange of

   A. mesons.
   B. hyperons.
   C. leptons.
   D. photons.

6. The deviations from the Bethe-Weizsäcker mass formula are observed for nuclei with

   A. no mass defect.
   B. magic decay length.
   C. extremely high binding energies.
   D. extremely low binding energies.

7. The basic idea of the nuclear shell model is that

   A. the nucleus has a shell structure on which the nucleons are fixed.
   B. in first approximation each nucleon moves independently in a common potential field.
   C. the spin-orbit interaction can be neglected.
   D. the nuclear potential has a sombrero shape.

8. The relevant interaction for the decay $n \rightarrow p + e^{-} + \overline{\nu}_e$ is the

   A. strong interaction.
   B. gravitational interaction.
   C. electrostatic interaction.
   D. weak interaction.

9. The indication for the existence of an almost mass-less, neutral particle (called neutrino) arises from the

   A. fast decay of certain nuclei species.
   B. continuous spectrum of the electron in the β decay.
   C. discrete spectrum of the electron in the β decay.
   D. excitation spectrum of β-particle emitting nuclei.

10. Which of the following particle properties is not necessarily conserved in particles reactions and decays?

    A. Strangeness
    B. Charge
    C. Baryon number
    D. Lepton number

Exercise 2.2: Radiocarbon dating (16 points)

Carbon-14 ($^{14}_{6}C$) is a β^- emitter with a half-life of 5730 years. It is formed in the upper atmosphere in collisions between cosmic rays of neutrons with nitrogen atoms ($^{14}_{7}N$) in the air. It is assumed that the fraction of the radioactive isotope ^{14}C to the total amount of C has remained constant in the atmosphere for the last 100 000 years. The current ratio in the atmosphere is ^{14}C/C = 1.33×10^{-12}.
During their life, plants maintain a constant ratio ^{14}C/C identical to that of the atmosphere. But when it dies, the exchange process with the air stops. The ^{14}C content in the plant will decrease with time because of the radioactive decay.
The molar mass of carbon is 12.011u. Other given constants: $N_A = 6.022\times10^{23}$/mol. The formula for radioactive decay is given by $N(t) = N(0)\, e^{-\lambda\, t}$.

a) (2 points) Write down the reaction of the formation of ^{14}C.
b) (2 points) Write down the reaction of the decay of ^{14}C.
c) (6 points) Calculate the activity (per gram of carbon) of atmospheric carbon due to the presence of ^{14}C.
d) (6 points) In analysing an archaeological specimen containing 500 mg of carbon, 174 decays are observed in one hour. What is the age of the specimen?

Exercise 2.3: Particle decays and conservation laws (18 points)

a) (4 points) Which of the following decays would you expect to have a longer lifetime? Why?

$$\pi^0 \rightarrow \gamma + \gamma \qquad \text{or} \qquad \pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

b) (2 points each) For each of the following processes, write down if it is allowed or not by checking conservation of energy, electric charge, lepton number, baryon number, strangeness.

1. $\Lambda^0 \rightarrow \pi^+ + \pi^-$
2. $\mu^+ \rightarrow e^+ + \nu_e + \nu_\mu$
3. $\bar{K}^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$
4. $p + \bar{\nu}_e \rightarrow n + e^+$
5. $K^- + p \rightarrow K^0 + K^+ + \Omega^-$

c) (4 points) Draw a Feynman diagram on the quark level of the decay $\Lambda^0 \rightarrow p + e^- + \bar{\nu}_e$.

Table 12-3 Hadrons that are stable against decay via the strong interaction

Name	Symbol	Mass (MeV/c^2)	Spin ($\hbar$)	Charge (e)	Antiparticle	Mean lifetime (s)	Typical decay products*
Baryons							
Nucleon	p (proton) or N^+	938.3	1/2	+1	$\bar{p}$	$>10^{32}$ y	
	n (neutron) or N^0	939.6	1/2	0	$\bar{n}$	930	$p + e^- + \bar{\nu}_e$
Lambda	Λ^0	1116	1/2	0	$\overline{\Lambda}{}^0$	2.5×10^{-10}	$p + \pi^-$
Sigma	Σ^+	1189	1/2	+1	$\overline{\Sigma}{}^-$	0.8×10^{-10}	$n + \pi^+$
	Σ^0	1192	1/2	0	$\overline{\Sigma}{}^0$	10^{-20}	$\Lambda^0 + \gamma$
	Σ^-	1197	1/2	−1	$\overline{\Sigma}{}^+$	1.7×10^{-10}	$n + \pi^-$
Xi†	Ξ^0	1315	1/2	0	$\overline{\Xi}{}^0$	3.0×10^{-10}	$\Lambda^0 + \pi^0$
	Ξ^-	1321	1/2	−1	$\overline{\Xi}{}^+$	1.7×10^{-10}	$\Lambda^0 + \pi^-$
Omega	Ω^-	1672	3/2	−1	Ω^+	1.3×10^{-10}	$\Xi^0 + \pi^-$
Charmed lambda	Λ_c^+	2285	1/2	+1	$\overline{\Lambda}{}_c^-$	1.8×10^{-13}	$p + K^- + \Lambda^+$
Mesons							
Pion	π^+	139.6	0	+1	π^-	2.6×10^{-8}	$\mu^+ + \nu_\mu$
	π^0	135	0	0	self	0.8×10^{-16}	$\gamma + \gamma$
	π^-	139.6	0	−1	π^+	2.6×10^{-8}	$\mu^- + \bar{\nu}_\mu$
Kaon	K^+	493.7	0	+1	K^-	1.24×10^{-8}	$\pi^+ + \pi^0$
	K^0	497.7	0	0	$\overline{K}{}^0$	0.88×10^{-10}	$\pi^+ + \pi^-$
						and	
						5.2×10^{-8}‡	$\pi^+ + e^- + \bar{\nu}_e$
Eta	η^0	549	0	0	self	2×10^{-19}	$\gamma + \gamma$

*Other decay modes also occur for most particles.
†The Ξ particle is sometimes called the cascade.
‡The K^0 has two distinct lifetimes, sometimes referred to as K^0_{short} and K^0_{long}. All other particles have a unique lifetime.

Table 12-11 Quark composition of selected hadrons

Baryons	Quarks	Mesons	Quarks
p	uud	π^+	$u\bar{d}$
n	udd	π^-	$\bar{u}d$
Λ^0	uds	K^+	$u\bar{s}$
Δ^{++}	uuu	K^0	$d\bar{s}$
Σ^+	uus	$\bar{K}^0$	$s\bar{d}$
Σ^0	uds	K^-	$s\bar{u}$
Σ^-	dds	J/ψ	$c\bar{c}$
Ξ^0	uss	D^+	$c\bar{d}$
Ξ^-	dss	D^0	$c\bar{u}$
Ω^-	sss	D_s^+	$c\bar{s}$
Λ_c^+	udc	B^+	$u\bar{b}$
Σ_c^{++}	uuc	$\bar{B}^0$	$\bar{d}b$
Σ_c^+	udc	B^0	$d\bar{b}$
Ξ_c^+	usc	B^-	$\bar{u}b$

Delta resonance states: Δ^-(ddd), Δ^0(udd), Δ^+(uud) and Δ^{++}(uuu)

Table 12-6 Some quantum numbers of the hadrons that are stable against decay via the strong interaction

Particle	Spin, $\hbar$	I	I_3	B	S	Y
p	1/2	1/2	+1/2	1	0	1
n	1/2	1/2	−1/2	1	0	1
Λ^0	1/2	0	0	1	−1	0
Σ^+	1/2	1	+1	1	−1	0
Σ^0	1/2	1	0	1	−1	0
Σ^-	1/2	1	−1	1	−1	0
Ξ^0	1/2	1/2	+1/2	1	−2	−1
Ξ^-	1/2	1/2	−1/2	1	−2	−1
Ω^-	3/2	0	0	1	−3	−2
π^+	0	1	+1	0	0	0
π^0	0	1	0	0	0	0
π^-	0	1	−1	0	0	0
K^+	0	1/2	+1/2	0	+1	+1
K^0	0	1/2	−1/2	0	+1	+1
η^0	0	0	0	0	0	0