Exam: Structure of Matter (NS-266B)
Subatomic Physics part

2 March 2017
Time: 09:30–12:30 (3 hours) – Please do not leave before 10:15).

The exam consists of two parts:
Part I tests knowledge in ten multiple-choice questions.
Part II consists of four open exercises.

The maximal number of points is indicated for each exercise.
The total number of points is 89 (plus 8 bonus points)

→ Answer each of the exercises on a separate piece of paper.
→ Write your name and student number on each page.
→ Do not give final answers only, explain your reasoning (short) and give full calculations.
→ Simple calculator use is allowed (no programmable).
→ No mobile/smart phone!

Good luck!

Part I – Multiple choice questions (20 points)

Instructions: Choose one answer. Each correct answer gives 2 points.

1. At current energies in the universe the nuclear force is

   A. a fundamental force.
   B. as strong as the electro-magnetic force.
   C. a residual of the electro-magnetic force.
   D. a residual of the strong force.

2. The cross section for a hard scattering (strong interaction) on a nucleus, scales with the type of nucleus via

   A. $A^{2/3}$
   B. A
   C. $Z^{2/3}$
   D. Z

3. The relatively large deviations from the Bethe-Weizsäcker mass formula are observed for nuclei with

   A. no mass defect.
   B. magic decay length.
   C. extremely high binding energies per nucleon.
   D. extremely low binding energies per nucleon.

4. The central assumption of the nuclear shell model is that

   A. the nucleus has a shell structure on which the nucleons are fixed.
   B. in first approximation each nucleon moves independently in a common potential field.
   C. the spin-orbit interaction can be neglected.
   D. the nuclear potential has a sombrero shape.

5. The existence of a neutral, almost mass-less particle (called neutrino) is evident from the

   A. fast decay of certain nuclei species.
   B. continuous spectrum of the electron in the β decay.
   C. discrete spectrum of the electron in the β decay.
   D. excitation spectrum of β-particle emitting nuclei.

6. The nuclear interaction between two nucleons can be considered as the exchange of

   A. mesons.
   B. gluons.
   C. leptons.
   D. photons.

7. Which statement is <u>wrong</u>? Elementary particles

   A. have no structure.
   B. are point-like objects.
   C. have excited states.
   D. are the building blocks of matter.

8. Which of the following particles have colour charge?

   A. Hadrons.
   B. Quarks.
   C. Leptons.
   D. Mesons.

9. The relevant particle property for the weak interaction is the

   A. baryon number.
   B. lepton number.
   C. electrical charge.
   D. quark flavour.

10. Which of the following particle properties <u>is not</u> necessarily conserved in particles reactions and decays?

    A. Strangeness.
    B. Charge.
    C. Baryon number.
    D. Lepton number.

Part II – Open questions

Exercise 1: Nuclear models (18 points)

The Weizsäcker empirical mass formula is $M(Z,A)c^2 = Zm_pc^2 + (A-Z)m_nc^2 - E_B$ where

$$E_B = a_1A - a_2A^{2/3} - a_3\frac{Z^2}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A} + a_5A^{-1/2}$$

and $a_1 = 15.7$ MeV, $a_2 = 17.2$ MeV, $a_3 = 0.72$ MeV, $a_4 = 93.2$ MeV, and $a_5 = (12, -12, 0, 0)$ MeV for even-even nuclei, odd-odd nuclei, odd-even nuclei and even-odd nuclei, respectively.

a) (4 points) Justify the first three terms in the formula for E_B and relate them to the idea of a liquid drop, which the model is based on.

b) (6 points) Use the empirical mass formula to find an expression for the Z values of the most stable isotopes of a particular A.

c) (4 points) Predict the binding energy of the last neutron (equal in magnitude to the single neutron separation energy) in ${}^{16}_{8}O$ using the Weizsäcker formula.

d) (4 points) What does it mean that this nuclide has a magic number of neutrons? List the 3 nuclides below in order of highest to lowest neutron separation energy (without calculating) and briefly explain your reasoning.

$${}^{15}_{8}O,\ {}^{16}_{8}O \text{ and } {}^{17}_{8}O$$

Exercise 2: Decay of heavy nuclei (10 points)

Consider the nucleus decay A → B + X. Particle A is initially at rest. Calculate the momenta (p_B and p_X) as well as the kinetic energy (E_B and E_X) from the released energy of the decay (= Q value). Calculate p and E_{kin} for the following decays (in eV related units):

a) (5 points) ${}^{238}U \rightarrow {}^{234}Th + \alpha$ (Q = 4.268 MeV)

b) (5 points) ${}^{57}Fe$ (E = 14.4 keV) → ${}^{57}Fe$ (ground state) + γ

Useful conversion factor: 1 u = 931 MeV/c^2 = $1.66054 \cdot 10^{-27}$ kg

Exercise 3: Conservation laws (18 points)

a) (2 points each) Check explicitly the following particle reactions/decays for violation of conservation of energy/mass, electric charge, baryon number, lepton number and strangeness number (by using the enclosed tables) and say whether they are allowed or forbidden and why:

A) $\pi^+ \rightarrow e^+ + \nu_e$

B) $\mu^- + n \rightarrow e^- + p$

C) $\pi^- + p \rightarrow \Delta^- + \pi^0$

D) $K^0 + p \rightarrow \Lambda^0 + K^0 + K^+$

E) $\Lambda^0 \rightarrow p + K^+$

b) (4 points each) Write down the quark content of each particle for the following particle reaction and decay (see enclosed tables) and draw the Feynman diagram <u>on quark level</u>:

F) $\pi^- + p \rightarrow K^+ + \Sigma^-$

G) $K^0 \rightarrow \pi^+ + \pi^-$

Exercise 4: Particle decays (31 points)

We consider the semi-leptonic decay of the K^+ (Kaon, quark content us) into a 'type of pion', 'a type of (anti-?)muon' and 'a type of (anti-?)neutrino'. In short, something that looks like:

$$K^+ \rightarrow \pi^? + \mu^? + (\nu_? \text{ or } \bar{\nu}_?)$$

a) (3 points) Use your knowledge of the conservation laws to complete the decay above. Show and explain stepwise that there is only one way to complete it.

If you did not find the correct decay in exercise (a), then use the following decay of the D^0 (quark content $c\bar{u}$) <u>for exercise (b) only</u>. Make sure to clearly state on your exam that you use this decay from now on: $D^0 \rightarrow \pi^- + e^+ + \nu_e$.

b) (4 points) Via which possible forces can this decay take place? Give an airtight argument why the other force(s) cannot result in this decay.

c) (6 points) Mention two different techniques by name to detect the muon, explain how they work, and explain how you would extract a momentum and/or energy from this measurement in not more than 30 words per technique.

Assume that the pion is produced at rest and that the neutrino is massless. The muon left the decay vertex in a circular curved motion with a radius of $r = 2$ meters in a magnetic field of $B = 1$ Tesla, and it has an angle of $\theta = \pi/4$ with the neutrino.

d) (6 points) Proof that p [MeV/c] = $300 \cdot q$[e (elementary charge unit)]$\cdot B$[Tesla]$\cdot r$[meter] and then calculate the momentum of the muon p_μ (MeV/c) and the energy E_μ (MeV) in three significant digits.

e) **(8 bonus points)** Show that, starting with an invariant mass calculation, we can express the momentum of the neutrino as

$$p_\nu = \frac{1}{c} \frac{c^4(m_K^2 - m_\pi^2 - m_\mu^2) - 2E_\mu m_\pi c^2}{2(E_\mu + m_\pi c^2 - c p_\mu \cos\theta)}$$

and calculate the momentum of the neutrino in two significant digits (Use 140 MeV/c^2 as the pion mass).

f) (4 points) Calculate the momentum of the initial Kaon in the laboratory frame in two significant digits. If you did not solve e), use 60 MeV/c for the momentum of the neutrino.