Midterm Exam

Structure of Matter

8-3-2022

Time: 13:30 – 15:30 (2 hours) – Please do not leave before 14:15.
There is extra time for people who are eligible for this.
The exam consists of two parts: Part I tests knowledge in multiple-choice questions.
Part II consists of open exercises.
The maximal number of points is indicated for each exercise.
Answer each of the exercises on a separate piece of paper.
Write your name and student number on each page.
Do not give final answers only, explain your reasoning (short) and give full calculations.
A simple calculator use is allowed (not programmable).
No mobile/smart phone!
Good luck!

1 10 Multiple Choice questions (10 points)

1. Which type of particle has 0 spin?

   A. Fermion

   B. Higgs

   C. Boson

   D. Lepton

2. The Rutherford's scattering law is true when the α-particle:

   A. penetrates the nucleus.

   B. undergoes a deep inelastic scattering.

   C. has a head-on collision on the gold nucleus.

   D. is absorbed by the gold nucleus.

3. The rest energy of an electron is of the order of:

   A. eV

   B. keV

   C. MeV

   D. GeV

4. Rutherford scattering does not depend on the ... of the incoming particle.

   A. electric charge

   B. kinetic energy

   C. impact parameter

   D. spin

5. In a β^+ decay an up quark becomes:

   A. a strange quark

   B. a down quark

   C. an anti-quark

   D. a top-quark

6. Via which decay will $^{14}_{6}C$ decay?

   A. α decay

   B. β^- decay

   C. β^+ decay

   D. γ decay

7. The main process by which energy is released in our Sun is called:

   A. fission

   B. Rutherford scattering

   C. fusion

   D. radioactivity

8. Which particles do not interact via the strong interaction?

   A. Baryons

   B. Leptons

   C. Mesons

   D. Fermions

9. Which Force acts on both leptons and quarks

   A. Strong Force

   B. Intermediate Force

   C. Weak Force

   D. Nuclear Force

10. What type of hadron is partially constructed of anti-quarks

   A. Baryons

   B. Leptons

   C. Mesons

   D. Photon

2 Open Questions: 40 points

2.1 Decays (10 points)

Consider we have the following decay chain:

- $N_1 \rightarrow N_2$ with a decay constant of $\lambda_1 = 10\ \mathrm{s}^{-1}$
- $N_2 \rightarrow N_3$ with a decay constant of $\lambda_2 = 50\ \mathrm{s}^{-1}$

We start with all the nuclei N_0 as N_1

a. **(2 points)** write the equation for $N_1(t)$

b. **(4 points)** Show that the equation for $N_2(t)$ can be written as:

$$N_2(t) = N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

c. **(4 points)** Draw the relative abundances of the N_1, N_2 and N_3 nuclei versus time, assuming N_3 does not decay (between 0 and 1 s).

2.2 Invariant Mass (10 points)

At the Large Hadron Collider (LHC) at CERN, proton-proton collisions are studied at the highest energy ever reached by humans. In one of these collisions two opposite sign muons ($m_\mu = 106$ MeV/c^2) are emitted. They go in opposite directions (back to back) and are having momenta of respectively 45 MeV/c and 30 GeV/c.

a. **(4 points)** Show that the relativistic relation

$$(M_{\text{inv}}c^2)^2 = E^2 - (|\mathbf{p}|c)^2, \tag{1}$$

where M_{inv} is the invariant mass, E the total energy of the system, $\mathbf{p}$ the total momentum, and c the speed of light, for two particles with mass m_1, m_2 and momentum $\mathbf{p_1}$, $\mathbf{p_2}$ can be written as:

$$\begin{aligned}(M_{inv}c^2)^2 &= (m_1c^2)^2 + (m_2c^2)^2 \\ &\quad + 2(E_1E_2 - |\mathbf{p_1}||\mathbf{p_2}|c^2 \cos\theta,\end{aligned} \tag{2}$$

where θ is the angle between the particles and E_1, E_2 their energies.

b. **(6 points)** If these two emitted muons in the collision originate from the decay of a particle what is the mass and momentum of this particle?

2.3 Center of Mass versus Laboratory frame (10 points)

Cosmic rays hitting the Earth are mostly protons having energies between approximately 10^8 and 10^{20} eV.

a. **(6 points)** At the LHC, protons hit each others at an energy of 13 TeV with opposite momenta, so that the laboratory system is in the Center of Mass System (CMS). What is the energy in eV to produce proton-proton interactions from cosmic rays equivalent to the collisions at the LHC?

b. **(2 points)** Why are these two energies so different?

c. **(2 points)** These highly energetic cosmic rays interact with the earth's atmosphere: describe which processes can happen, which are the particles we typically observe at sea-level and how we can detect them.

2.4 Reactions and Feynman diagrams (10 points)

Check the following particle reactions and decays for violation of the conservation of energy/mass, electric charge, baryon number, lepton number and strangeness number (use the enclosed tables) say whether they are allowed or forbidden and via which force the reaction happens:

a) **(1 points)** $\mu^- \to \overline{\nu_\mu} + e^- + \overline{\nu_e}$

b) **(1 points)** $\pi^0 + p \to n + p$

c) **(1 points)** $e^- + p \to n + \nu_e$

d) **(1 points)** $D^+ \to K^- + \pi^+ + \pi^+$

e) **(1 points)** $p \to p + K^- + K^+$

f) **(1 points)** $J/\Psi \to \mu^+ + \mu^-$

Write down the Feynman diagrams on quark level for the following particle reactions (for the quark content of each particle see the enclosed tables):

g) **(2 points)** $g + u \to b + \overline{b} + u$

h) **(2 points)** $K^+ \to \pi^+\pi^+\pi^-$

Table 12-11 Quark composition of selected hadrons

Baryons	Quarks	Mesons	Quarks
p	uud	π^+	$u\bar{d}$
n	udd	π^-	$\bar{u}d$
Λ^0	uds	K^+	$u\bar{s}$
Δ^{++}	uuu	K^0	$d\bar{s}$
Σ^+	uus	$\bar{K}^0$	$s\bar{d}$
Σ^0	uds	K^-	$s\bar{u}$
Σ^-	dds	J/ψ	$c\bar{c}$
Ξ^0	uss	D^+	$c\bar{d}$
Ξ^-	dss	D^0	$c\bar{u}$
Ω^-	sss	D_s^+	$c\bar{s}$
Λ_c^+	udc	B^+	$u\bar{b}$
Σ_c^{++}	uuc	$\bar{B}^0$	$\bar{d}b$
Σ_c^+	udc	B^0	$d\bar{b}$
Ξ_c^+	usc	B^-	$\bar{u}b$

Table 12-6 Some quantum numbers of the hadrons that are stable against decay via the strong interaction

Particle	Spin, ħ	I	I_3	B	S	Y
p	1/2	1/2	+1/2	1	0	1
n	1/2	1/2	−1/2	1	0	1
Λ^0	1/2	0	0	1	−1	0
Σ^+	1/2	1	+1	1	−1	0
Σ^0	1/2	1	0	1	−1	0
Σ^-	1/2	1	−1	1	−1	0
Ξ^0	1/2	1/2	+1/2	1	−2	−1
Ξ^-	1/2	1/2	−1/2	1	−2	−1
Ω^-	3/2	0	0	1	−3	−2
π^+	0	1	+1	0	0	0
π^0	0	1	0	0	0	0
π^-	0	1	−1	0	0	0
K^+	0	1/2	+1/2	0	+1	+1
K^0	0	1/2	−1/2	0	+1	+1
η^0	0	0	0	0	0	0

Table 12-3 Hadrons that are stable against decay via the strong interaction

Name	Symbol	Mass (MeV/c^2)	Spin ($\hbar$)	Charge (e)	Antiparticle	Mean lifetime (s)	Typical decay products*
Baryons							
Nucleon	p (proton) or N^+	938.3	1/2	+1	$\bar{p}$	$>10^{32}$ y	
	n (neutron) or N^0	939.6	1/2	0	$\bar{n}$	930	$p + e^- + \bar{\nu}_e$
Lambda	Λ^0	1116	1/2	0	$\bar{\Lambda}^0$	2.5×10^{-10}	$p + \pi^-$
Sigma	Σ^+	1189	1/2	+1	$\bar{\Sigma}^-$	0.8×10^{-10}	$n + \pi^+$
	Σ^0	1192	1/2	0	$\bar{\Sigma}^0$	10^{-20}	$\Lambda^0 + \gamma$
	Σ^-	1197	1/2	−1	$\bar{\Sigma}^+$	1.7×10^{-10}	$n + \pi^-$
Xi†	Ξ^0	1315	1/2	0	$\bar{\Xi}^0$	3.0×10^{-10}	$\Lambda^0 + \pi^0$
	Ξ^-	1321	1/2	−1	$\bar{\Xi}^+$	1.7×10^{-10}	$\Lambda^0 + \pi^-$
Omega	Ω^-	1672	3/2	−1	$\bar{\Omega}^+$	1.3×10^{-10}	$\Xi^0 + \pi^-$
Charmed lambda	Λ_c^+	2285	1/2	+1	$\bar{\Lambda}_c^-$	1.8×10^{-13}	$p + K^- + \Lambda^+$
Mesons							
Pion	π^+	139.6	0	+1	π^-	2.6×10^{-8}	$\mu^+ + \nu_\mu$
	π^0	135	0	0	self	0.8×10^{-16}	$\gamma + \gamma$
	π^-	139.6	0	−1	π^+	2.6×10^{-8}	$\mu^- + \bar{\nu}_\mu$
Kaon	K^+	493.7	0	+1	K^-	1.24×10^{-8}	$\pi^+ + \pi^0$
	K^0	497.7	0	0	$\bar{K}^0$	0.88×10^{-10} and 5.2×10^{-8}‡	$\pi^+ + \pi^-$; $\pi^+ + e^- + \bar{\nu}_e$
Eta	η^0	549	0	0	self	2×10^{-19}	$\gamma + \gamma$

*Other decay modes also occur for most particles.
†The Ξ particle is sometimes called the cascade.
‡The K^0 has two distinct lifetimes, sometimes referred to as K^0_{short} and K^0_{long}. All other particles have a unique lifetime.

Lepton masses:

$m_{\text{electron}} = 0.511$ MeV/c^2
$m_{\text{muon}} = 106$ MeV/c^2
$m_{\text{tau}} = 1.777$ GeV/c^2

Other Mesons:

J/Ψ: $m(J/\Psi) = 3097$ MeV/c^2: quark content $c\bar{c}$
D^+: $m(D^+) = 1870$ MeV/c^2: quark content $c\bar{d}$