Examination Particle Physics I

Wednesday January 6th 2016

- This exam has four exercises.
- Make all exercises on a separate piece of paper.
- Write your name on each piece of paper that you hand in.
- Write your student number on the first paper.
- Write in a readable way. It is better to hand in readable answers with mistakes than unreadable answers that are correct.

1 Theory questions (2.5 points)

(a.) Fermi's Golden Rule

(i) Write down (don't derive!) the expression for the differential cross-section $d\sigma/d\Omega$ of two-body scattering ($AB \to CD$) in the center of momentum frame. Express your answer in the invariant amplitude $\mathcal{M}$, and the momenta of the initial state and final state particles.

(ii) Similarly, write down the expression for the partial decay width of a two-body decay ($A \to BC$) in the rest frame of particle A.

(b.) Feynman rules

(i) Draw the t-channel diagram for two particles interacting by the exchange of a single force carrier.

(ii) Ignoring the mass of the incoming and outgoing particles, write down (don't derive!) the expression for the angular dependence of the t-channel cross-section in the centre-of-momentum system in case the exchanged particle has spin 1.

(iii) Similarly, write down that expression in case the exchanged particle has spin 0.

(iv) Give an example of a spin-0 and a spin-1 force carrier in the Standard Model.

(c.) The Standard Model

(i) Which local gauge symmetries are implemented in the Standard Model lagrangian and what are the related quantum numbers?

(ii) What do we mean if we talk about 'charge screening'?

(iii) The coupling constant of the electromagnetic interaction has a different dependence on the distance between charges than the couplings of the weak and strong interactions. How is this difference related to the gauge symmetry?

2 B mesons at PEP-II (2.75 points)

The PEP-II collider at SLAC is an e^+e^- collider. The beam energies of the collider are tuned such that the centre-of-momentum energy is just above the threshold for $b\bar{b}$ production, at the so-called $\Upsilon(4S)$ resonance, $\sqrt{s} = 10.580$ GeV.

(a.) Draw the leading order Feynman diagram for the production of a $b\bar{b}$ quark pair at PEP-II.

(b.) As quarks are not free particles, what is actually produced are two B mesons, mesons that consist of a (anti-) b quark and an (anti-) u or d quark. Write down the quark content of the B^+ meson and of the B^0 meson. (By convention the B^0 meson contains an anti-b quark.)

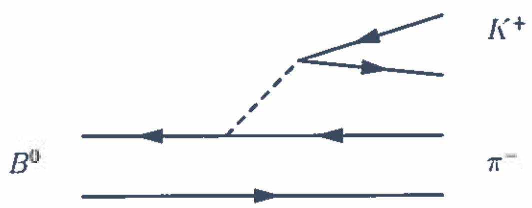
For the physics done at the Babar experiment at PEP-II it is important to measure the B meson decay time. B mesons have an average lifetime τ_B of about 1.5 pico-second (ps) and a mass of about 5.27 GeV/c^2. The combined mass of the two B mesons is just below the $\Upsilon(4S)$ resonance.

If the two beams of the PEP-II collider were tuned to the same energy, the produced B mesons would be practically at rest, making it impossible to measure their decay time. Therefore, PEP-II is an *asymmetric* collider: the positron beam has a different energy than the electron beam, such that the two B mesons are boosted: they fly with a common velocity in the same direction after they are produced.

(c.) Given that the positron beam has an energy of $E_+ = 3.1$ GeV, compute the energy of the electron beam. You may ignore the electron and positron mass. You do not need to give a numerical value: express the answer in terms of E_+ and $\sqrt{s}$.

(d.) Compute the average distance a B meson will travel at PEP-II before it decays. You may ignore the velocity of the B meson in the e^+e^- rest frame.

(e.) One decay modes of the B^0 meson is to a K^+ and a π^-. Considering that it is the b quark that decays, assign the name of the quarks to each of the external lines in the following diagram for this decay (time increases from left to right):



(f.) The disjoint (bottom) line is called the spectator quark. Ignoring the spectator quark and assuming free quarks, write down the invariant amplitude $\mathcal{M}$ for the decay.

(g.) Figure 1 (below) shows the invariant mass distribution for $K^+\pi^-$ and $K^-\pi^+$ candidates observed at the BaBar experiment at PEP-II. BaBar observes a difference in the number of B^0 and $\bar{B}^0$ particles in this decay mode. This is evidence for CP violation. What is the origin of CP violation in the Standard Model?

(h.) Another decay mode of the B^0 meson is to a D^- $(d\bar{c})$ and π^+ $(u\bar{d})$. Ignoring the mass of the u, d, s and c quarks, estimate the ratio of the $B^0 \to K^+\pi^-$ and $B^0 \to D^-\pi^+$ branching ratios in terms of elements of the CKM matrix.

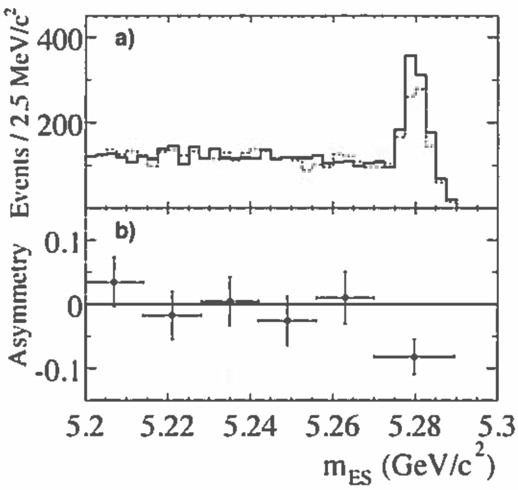


Figure 1: Invariant mass distribution for $K^+\pi^-$ (blue solid) and $K^-\pi^+$ (red dashed) candidates observed in the BaBar experiment at SLAC. The peak on the right corresponds to B meson decays in these final states. The tail below the peak is due to other incompletely reconstructed B meson decays and 'continuum' production. (From Phys.Rev.Lett. 93 (2004) 131801.)

3 An alternative Higgs potential (2.25 points)

Note: apart from question c), these exercises require only very little computation.

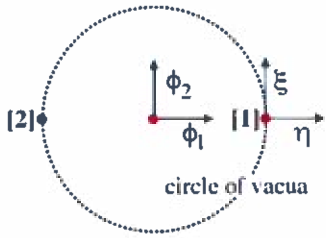
In the lecture we discussed the Higgs mechanism in the Standard Model. Using explicitly the normalisation factor $\frac{1}{\sqrt{2}}$ in the definition of the field ϕ the potential terms in the Lagrangian related to the Higgs field are given by:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad \text{with } \mu^2 < 0 \text{ and } \lambda > 0 \tag{1}$$

After going to the vacuum ϕ_0, using the unitary gauge and looking at perturbations around the minimum we see that the Higgs boson mass is given by $m_h = \sqrt{2\lambda v^2}$.

part 1: Same potential, but different vacuum

(a.) Consider the figure on the right. In the unitary gauge we described perturbations using the fields η and ξ. What changes if we would take [2] as the vacuum instead of [1]? Is the physics the same ?



(b.) How do you see from the full Lagrangian that the Higgs boson is a scalar particle?

part 2: New potential: different masses for particles

Imagine that the second term in the potential would be a ϕ^6-term. We would then have:

$$V_{\rm new}(\phi) = \alpha^2\phi^2 + \beta\phi^6 \qquad \text{with } \alpha^2 < 0 \text{ and } \beta > 0 \tag{2}$$

(c.) What is the value of ϕ that minimises this potential: $\phi_0 = v_{new}$? What is the predicted mass of the Higgs boson in terms $v_{\rm new}$ and β?

(d.) What are the masses of the W^+ and W^- gauge bosons in terms of $v_{\rm new}$ and β?

(e.) Explain why there is no higher order term than ϕ^4 in the SM Higgs potential.

4 The Higgs boson at the LHC (1.5 points)

Note: these exercises require only very little computation.

One of the signs for the existence of the Higgs boson at the Large Hadron Collider was the observation of a mass peak in the channel where the Higgs boson decays into four muons:

$$pp \to H \to ZZ^* \to \mu^+\mu^-\mu^+\mu^-$$

In the experiment we measure for each of the muons (μ_i), its energy E_i and momentum p_i.

(a.) Starting from the four measured energies and momenta, how can you estimate the mass of the Higgs boson in a particular event?

(b.) With $m_h = 125$ GeV and $m_Z = 91.2$ GeV, how can the Higgs decay into two Z bosons?

The Higgs boson can also decay into other particles. The decay width is mainly determined by the mass of the particle and that of the Higgs boson. The branching fractions are shown as a function of the Higgs mass in the left figure below.

(c.) Why is there a 'dip' in the branching fraction to ZZ* for a Higgs boson mass around 160 GeV?

There is also a possibility to produce two Z bosons from other (SM) processes, for example from quark annihilation as is shown in the right figure below.

(d.) Draw the correct leading order Feynman diagram for this process in the Standard Model.

(e.) Do you expect a difference in the angular distributions of the four muons between this channel and the one originating from Higgs decay? Why (not)?