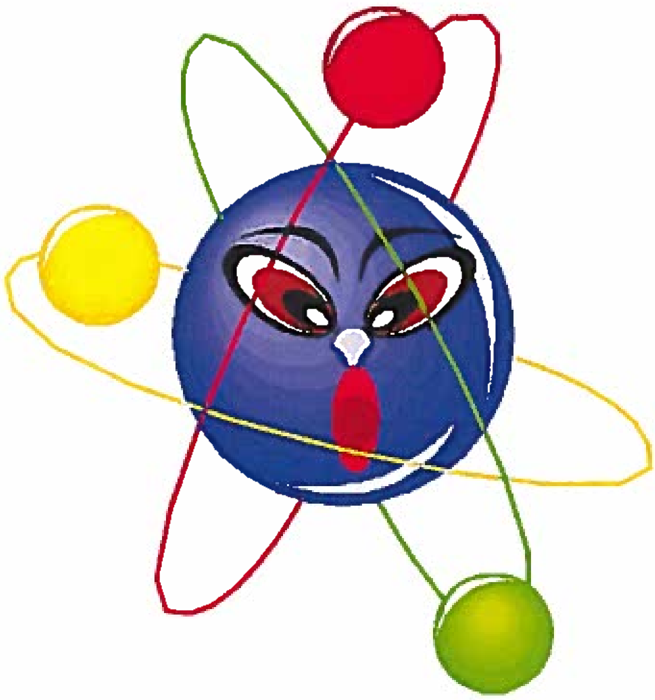
Examination
Experimental Quantum Physics
Part Two



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Zeeman slowing: The force on an atom in an electro-magnetic field is given by

$$\vec{F} = \hbar\vec{k}\gamma_p = \frac{\hbar\vec{k}s_0\gamma/2}{1 + s_0 + (2\delta_\pm/\gamma)^2}.$$

1. Describe in your own words the symbols $\hbar\vec{k}$, γ_p, s_0, γ and δ.
2. What is the maximum force and acceleration on resonance?

   For Zeeman slowing the detung δ is given by

   $$\delta' = \delta - \vec{k}\cdot\vec{v} - \frac{\mu' B}{\hbar},$$

   with μ' the difference between the magnetic moment of the excited and ground state.
3. Find the maximum field B at the beginning of the slower required to slow atoms down from 1000 m/s. Assume $\mu' = \mu_B$.
4. Find the minimum length required to slow down atoms from 1000 m/s to standstill.
5. Describe the difference between laser slowing and laser cooling.
6. Indicate, if in the case of Zeeman slowing there is only slowing, or also cooling.

Important numbers: $m_{\mathrm{Na}} = 23$ amu, 1 amu $= 1.67 \times 10^{-27}$ kg, $\lambda = 589$ nm, $\mu_B = 9.27 \times 10^{-28}$ J/G and $\hbar = 1.05 \times 10^{-34}$Js.

Bose-Einstein condensation: The Maxwell-Boltzmann distribution function for a classical gas is given by

$$f_{\mathrm{MB}}(\epsilon) = \exp[-(\epsilon - \mu)/k_B T].$$

1. What is the meaning of the symbol μ? Describe in your own words, how it is used in thermodynamics.

   For an uniform gas the density of states is given by

$$\rho(\epsilon) = \frac{V}{4\pi^2\hbar^3}\sqrt{8m^3\epsilon}.$$

2. Calculate the number of atoms at a certain temperature T in a volume V. Express your result in terms of the deBroglie wavelength $\lambda_{\mathrm{deB}} = h/\sqrt{2\pi m k_B T}$. *Hint: Use the definite integral given below this exercise to evaluate your integral.*
3. Use this result to determine μ and discuss its limits for small and large atom densities.
4. What happens, when $N/V = 1/\lambda_{\mathrm{deB}}^3$? What happens if $N/V > 1/\lambda_{\mathrm{deB}}^3$ and do you consider this range physically sound?
5. Determine the total energy E of the system and express your result in terms of the total number of atoms N.

   In an isotropic, harmonic trap the density of states becomes

$$\rho(\epsilon) = \frac{\epsilon^2}{2\hbar^3\omega^3}$$

   with ω the trap frequency.

6. Find the total number of atoms N and the total energy of the system E in an harmonic trap.
7. Compare your results for the energy per particle in 5) and 6) and discuss the difference.

   For a Bose gas the distribution function is given by

$$f_{\mathrm{BE}}(\epsilon) = \frac{1}{\exp[(\epsilon - \mu)/k_B T] - 1}.$$

8. Discuss under which conditions this distribution function becomes comparable to the Maxwell-Boltzmann distribution function.
9. Does the Bose-Einstein statistics lead to a larger or smaller occupation of the lowest states compared to Maxwell-Boltzmann statistics. Indicate how this can be explained using the Dirac symmetrization principle.

The following definite integral can be used, if necessary:

$$\int_0^\infty \mathrm{d}x \; x^{\alpha-1} e^{-x} = \Gamma(\alpha).$$

For various values of α the values are

α	$\Gamma(\alpha)$
1	1
3/2	$\sqrt{\pi}/2$
2	1
5/2	$3\sqrt{\pi}/4$
3	2
7/2	$15\sqrt{\pi}/8$
4	6

The Zeeman effect:

The interaction of an atom with the magnetic field is given by

$$\mathcal{H}_Z = -\vec{\mu} \cdot \vec{B},$$

with $\vec{\mu}$ the magnetic dipole moment. For an atom the magnetic moment is given by

$$\vec{\mu} = -\frac{\mu_B(\vec{\ell} + 2\vec{s})}{\hbar},$$

with $\mu_B = e\hbar/2m$ the Bohr magneton, $\vec{\ell}$ the orbital angular momentum and $\vec{s}$ the spin angular momentum. We want to calculate the effect of the Zeeman effect for the p-state of the alkali-metal atoms.

1. Give the quantum numbers ℓ and s of the p-state.
2. For a strong magnetic field, calculate the Zeeman shift for the p-state assuming the magnetic field is in the z-direction.

   The states are split by the spin-orbit interaction, which is given by

   $$\mathcal{H}_{\text{SO}} = \frac{\xi(r)\vec{\ell} \cdot \vec{s}}{\hbar^2},$$

   where the function $\xi(r)$ is only a function of the radius r of the electron. In this exercise we will only use $A = \langle \xi(r) \rangle$.
3. What are the possible values of the quantum number j for the p-state, where $\vec{j}$ is the total angular momentum: $\vec{j} = \vec{\ell} + \vec{s}$.

   Hint: In this exercise we will neglect the effect of the nuclear spin I, since the shift caused by the nuclear spin is much smaller compared to the spin-orbit shift.
4. Show that $\vec{\ell} \cdot \vec{s}$ can be written as

   $$\vec{\ell} \cdot \vec{s} = \frac{1}{2}\left(\ell_+ s_- + \ell_- s_+\right) + \ell_z s_z,$$

   In the p-state of the alkali-metal atoms there are six magnetic substates, which are given in the uncoupled basis by

state	m_ℓ	m_s
1	-1	$-1/2$
2	0	$-1/2$
3	$+1$	$-1/2$
4	-1	$+1/2$
5	0	$+1/2$
6	$+1$	$+1/2$

5. Show that the spin-orbit interaction does not change $m_\ell + m_s$. For $m_\ell + m_s = +1/2$ the 2×2 submatrix for the Hamiltonian is given by

$$\mathcal{H}' = \mathcal{H}_{\rm SO} + \mathcal{H}_Z = \begin{pmatrix} -A/2 & +A/\sqrt{2} \\ +A/\sqrt{2} & +\mu_B B \end{pmatrix},$$

with $A = \langle \xi(r) \rangle$.

6. Determine the eigenvalues for these two states.

7. At which strength of the magnetic field B is the spin-orbit interaction equal to the Zeeman interaction? Discuss your result.