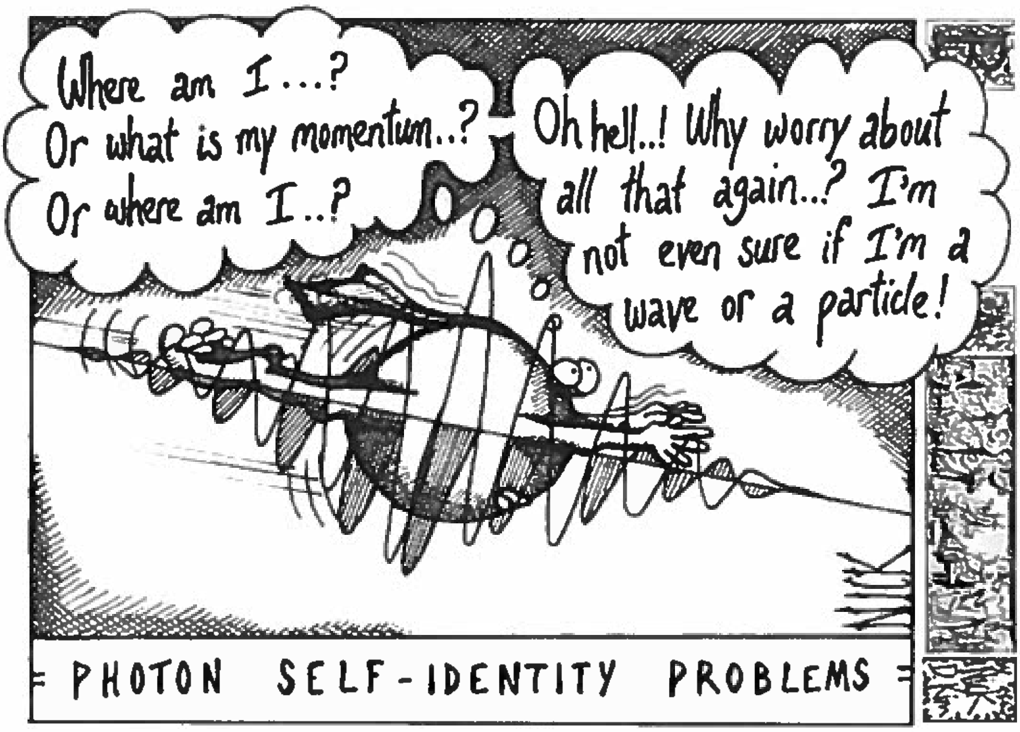
First Test Photon Physics



Peter van der Straten

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1. Dressed states

In this problem we will describe the amplitude of the excited and ground state by a spinor $\chi = (\tilde{c}_e, c_g)$. This way we can treat the two-level atom as an effective spin-1/2 system. The evolution equations is given by

$$i\hbar \frac{\mathrm{d}\chi}{\mathrm{d}t} = \mathcal{H}_{\text{eff}}\chi,$$

with

$$\mathcal{H}_{\text{eff}} = \frac{\hbar}{2}\begin{pmatrix} -2\delta & \Omega \\ \Omega & 0 \end{pmatrix}.$$

Here we assume the Rabi frequency Ω to be real.

a) Determine the eigenvalues of $\mathcal{H}_{\text{eff}}$.

b) The eigenstates are required to be normalized and orthogonal. Why do the following eigenspinors

$$|1\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \qquad \text{and} \qquad |2\rangle = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix},$$

fulfill these two requirements?

c) Determine the mixing angle θ.

d) Determine the eigenvalues for $\delta \gg \Omega$.

e) In which states are the atoms under this condition predominantly in these two eigenstates?

2. Photon echo

In this exercise we consider photon echoes. In a condensed-matter system the absorption frequency of the active atoms is considerably broadened due to the environment of the individual atoms. So each atom has its own absorption frequency and thus its own detuning with respect to the laser frequency. In order to negate the effects of the inhomogeneities, the "photon echo" technique has been developed. First, a $\pi/2$-pulse is applied, after a waiting period τ followed by a π-pulse, and after another waiting period τ followed by a $\pi/2$-pulse. Here we will use the Bloch picture to describe the action of the radiation field. The evolution of the Bloch vector $\vec{R} = (u, v, w)$ is given by

$$\frac{\mathrm{d}\vec{R}}{\mathrm{d}t} = \vec{\Omega} \times \vec{R},$$

with the vector $\vec{\Omega} = (-\Omega_r, -\Omega_i, -\delta)$ describing the action of the field. Here we will choose the Rabi frequency to be real.

a) Assuming all atoms are initially in the ground state, what will the Bloch vector of the atoms be after the first $\pi/2$-pulse, where the time-duration of the $\pi/2$-pulse is given by $t_{\pi/2} = \pi/2\Omega$. Keep in mind, that the atoms are inhomogeneously broadened and thus all have different resonance frequencies. Explain your answer.

b) Explain under which condition all the atoms will be nearly on the equator?

Here we will assume that the Bloch vector of all atoms after the first $\pi/2$-pulse be on the equator.

c) Describe what happens to the Bloch vectors during the first waiting period τ? Assume $\tau \gg t_{\pi/2}$.

d) What happens to the Bloch vectors during the π-pulse? Use the same condition as found under b).

e) Describe what happens to the Bloch vectors during the second waiting period τ?

f) What happens to the Bloch vectors during the second $\pi/2$-pulse.

g) **Bonus:** Why is the technique called "photon echo"?