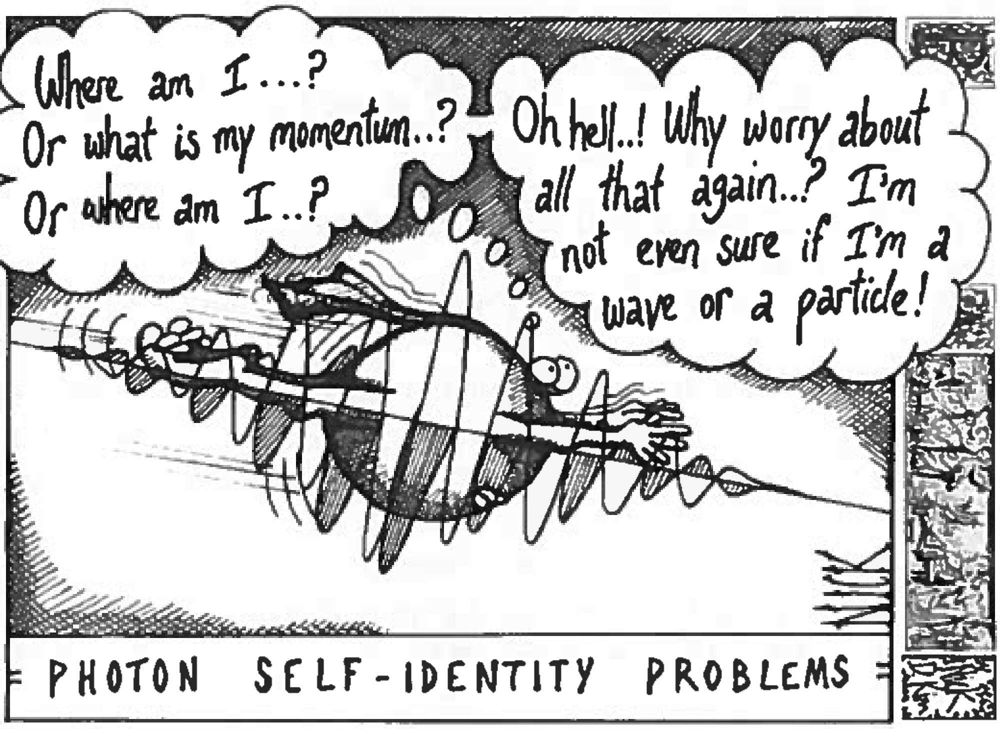
Second Test Photon Physics



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1. Gausian beams

The electric field of a Gaussian laser beam is given by:

$$E_0(\vec{r}) = \frac{Ae^{i\phi(z)}}{\sqrt{1+\left(\frac{z}{z_R}\right)^2}} \exp\left(\frac{ik\rho^2}{2R(z)}\right) \exp\left(\frac{-\rho^2}{w^2(z)}\right),$$

with $\rho^2 = x^2 + y^2$, $w(z) = w_0\sqrt{1+(z/z_R)^2}$, $z_R = \pi {w_0}^2/\lambda$, $R(z) = z + {z_R}^2/z$, and $\phi(z) = \arctan(z/z_R)$

a) Describe in your own words the physical meaning of $R(z)$, $w(z)$, w_0, z_R and ϕ.

b) Calculate the intensity $I(x, y, z)$ of the beam given by $I(x, y, z) = \varepsilon_0 c|E(x, y, z)|^2/2$.

c) Show that the power of the beam (*i.e.*, the intensity integrated over the radial direction) does not depend on z, which you expect from conservation of energy.

d) Determine the electric field at large distances ($z \gg z_0$).

e) Show why this field has the form of a spherical wave with its center of curvature located at the beam waist ($z = 0$).

2. Gaussian beam in a fiber

The index of refraction in the central part of a fiber (=core) is given by

$$n(r) = n_0(1 - r^2/2\ell^2),$$

with r the distance to the center of the fiber and $\ell \approx 10^{-2}$ cm a material constant. The core has a diameter of 50 μm. For a light ray impinging on the fiber we have the following relation for the distance $r(z)$ to the center of the fiber:

$$r(z) = r_0 \cos\frac{z}{\ell} + r_0'\ell \sin\frac{z}{\ell},$$

with (r_0, r_0') the ray vector at the beginning of the fiber and z the distance in the fiber.

a) Prove that the ABCD-matrix M for this fiber is given by

$$M = \begin{pmatrix} \cos\frac{z}{\ell} & \ell \sin\frac{z}{\ell} \\ \frac{-1}{\ell}\sin\frac{z}{\ell} & \cos\frac{z}{\ell} \end{pmatrix}$$

Show that the determinant of this matrix is 1.

The q-parameter is given by

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{i\pi\lambda}{w(z)^2}.$$

Considering the length of fibers (many kilometers) the only possible Gaussian beam in the fiber has a q-parameter, that is independent of z.

b) Calculate the q-parameter of this beam.

c) Calculate the waist w of this beam.

d) Calculate for visible light ($\lambda = 600$ nm) the ratio between the intensity at the edge of the core and in the center of the core.

e) Show that the losses due to the finite size of the core are small.