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Particle Physics II

Exam

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Every sub-question has the same absolute weight.

Exercise 1

CP violation in the Standard Model and the CKM-matrix

In the early '60's it was known that the transition amplitude between electrons and electron neutrinos is larger than the transition amplitude between up and down quarks. The transition between up and strange quarks is even smaller.

(a) In 1963 Cabibbo parametrized the magnitude of these quark couplings with one parameter. In 1970 Glashow, Iliopoulis and Maiani postulated a fourth quark, introducing the quark-mixing matrix for 2×2 quarks. If we require that this complex matrix is unitary, how many free parameters does it contain?

(b) How would you parametrize this matrix? Explain why the expression of "rotated quark fields" is used. Argue why Wolfenstein chose the introduction of λ in his parametrization and deduce the expression of V_{ud}, starting from V_{us}:

$$V_{2\times 2} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda \\ -\lambda & 1-\frac{1}{2}\lambda^2 \end{pmatrix} \tag{0.0.1}$$

(c) What is the phase difference between $V_{us}^* V_{cs}$ and $V_{ud}^* V_{cd}$?

Exercise 2

Announcement of CP violation in charm decays

In 1964 CP violation (in mixing) was discovered in the kaon system. In 2003 CP violation (in the interference of mixing and decay) was discovered in the B-system.
Tomorrow the discovery of CP violation in the charm-system will be announced! Let's inspect the decays of neutral D^0 mesons in a pair of charged mesons, and study this historic event. The D^0 meson is the ground state (S=0) of the $(c\bar{u})$ quark combination and has a mass of about 1.86 GeV.

(a) The D mesons have smaller lifetimes compared to B mesons, despite the fact that the B-meson is about 2.8× heavier than the D-meson. Heavier particles usually decay faster than lighter particles due to the available phase space for the decay products.
What is the reason for the relatively long B lifetime?

(b) D^0 mesons can oscillate into $\bar{D}^0$ mesons and vice versa. If you consider the box-diagram, which internal quarks can contribute to $D^0-\bar{D}^0$ mixing?

(c) For each possible internal quark, give the corresponding CKM-elements, and estimate the corresponding amplitude in terms of orders of the Wolfenstein parameter λ.

(d) Do you expect faster or slower oscillations compared to the neutral B-meson oscillations? Why?

(e) Do you expect to measure CP-violation in the intereference of mixing and decay with D^0 and/or $\bar{D}^0$ decays? Why (not) ?

(f) Draw the simplest tree (T) Feynman diagram for the decay $D^0 \to K^+K^-$, and indicate the relevant CKM-elements.

(g) Show that $|K^+K^- >$ is a CP-eigenstate, and give the CP-eigenvalue.

(h) Let's investigate the possibility of CP-violation in the decay of $D^0 \to K^+K^-$. For this, we need a second decay amplitude. Draw a penguin (P) diagram for this decay, and indicate the relevant CKM elements.

(i) If you assume a strange quark as internal quark, how do the T and P amplitudes compare in magnitude, in orders of the Wolfenstein parameter λ?

(j) What is the weak phase difference between the T and P amplitude?

(k) Repeat the exercise (i) above, but now assuming the bottom quark as internal quark.

(l) Do you expect large CP violation in decay for the scenario in (i)? Why (not)?

(m) Do you expect large CP violation in decay for the scenario in (k)? Why (not)?

The CP asymmetry for this decay is expressed as

$$A_{CP} = \frac{\Gamma(D^0 \to K^+K^-) - \Gamma(\bar{D}^0 \to K^+K^-)}{\Gamma(D^0 \to K^+K^-) + \Gamma(\bar{D}^0 \to K^+K^-)}$$

An experimental difficulty is to determine whether the decaying D meson was a D^0 or $\bar{D}^0$ meson. A powerful method is to select D mesons originating from the decays $D^{*+} \to D^0\pi^+$ and $D^{*-} \to \bar{D}^0\pi^-$, such that the charge of the accompanying pion tells you the flavour of the neutral D meson under study. However, in pp collisions, the amount of produced D^{*-} mesons might differ from the amount of D^{*+} mesons, affecting the *measured* asymmetry.

An extremely accurate measurement can be done by comparing the CP asymmetry of $D \to K^+K^-$ decays to the CP asymmetry of $D \to \pi^+\pi^-$ decays,

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$$

(n) Draw the tree (T) diagram of $D^0 \to \pi^+\pi^-$ decays.

(o) Draw the penguin (P) diagram of $D^0 \to \pi^+\pi^-$ decays.

(p) What is the relative phase between the T diagrams in (m) and (f) ?

(q) Charm decays are generally difficult to interpret, because long-distance QCD effects (such as *rescattering* where the final state $\pi^+\pi^-$ might change into K^+K^-, when the $d\bar{d}$ pair changes into an $s\bar{s}$ through gluon exchange). As a result, our diagrammatic approach is too simplistic.
Nevertheless, if we would assume perfect SU(3) symmetry (i.e. the symmetry when exchanging all d and s quarks), do you expect $A_{CP}(K^+K^-)$ and $A_{CP}(\pi^+\pi^-)$ to cancel?

The measurement that will be announced tomorrow at the famous Moriond conference is

$$\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$$

with 5.4σ significance.

Exercise 3

Transformations, group theory and the colour quantum number

(a) Consider the operator $\hat{U} = e^{-i\alpha\hat{F}/\hbar}$, where α is a displacement in real space along the x axis and $\hat{F}$ the generator of the transformation. Identify the symmetry generator of the transformation if the Hamiltonian of the system is invariant under translation in space and indicate what is the conserved quantity.

(b) Consider two particles of spin 1 each in a state $S_z = -1$ (the first) and $S_z = 1$ (the second). What is the probability of each state of the total angular momentum if the orbital angular momentum is 0. For this, you can consult the table of figure 0.2.

(c) Considering the quark contents of known hadrons, argue why there was a need to introduce a new quantum number i.e. the one of colour.

(d) QCD allows exotic hadron states to exist, such a tetra- and penta-quarks. In which colour state would you expect to discover experimentally such states?

(e) Why is QCD described by SU(3) and not by the U(3)-colour group?

(f) Associate each gluon state to the relevant Gell-Mann matrix and explain the association.

(g) How do the gluons manifest their existence in elementary processes such as $e^+ + e^-$?

(h) Calculate the colour factors for the following state transition: $(BG+GB)/\sqrt{2} \rightarrow (BG+GB)/\sqrt{2}$

(i) Are the colour interactions between a quark and an antiquark attractive or repulsive and why?

For these you have to consider that the colour matrices are given by:

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \text{ for R, } \begin{pmatrix}0\\1\\0\end{pmatrix} \text{ for B, } \begin{pmatrix}0\\0\\1\end{pmatrix} \text{ for G}$$

The Gell-Mann matrices are:

$$\underbrace{\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}}_{\lambda_1} \underbrace{\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}}_{\lambda_2} \underbrace{\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}}_{\lambda_3} \underbrace{\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}}_{\lambda_4}$$

$$\underbrace{\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}}_{\lambda_5} \underbrace{\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}}_{\lambda_6} \underbrace{\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}}_{\lambda_7} \underbrace{\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}}_{\lambda_8}$$

Exercise 4

The QCD Lagrangian, the running coupling constant and DIS

(a) The QCD Lagrangian in the Standard Model is given by

$$\mathscr{L}_{\text{QCD}} = -\tfrac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ - m)\psi - g_s\overline{\psi}\gamma_\mu A^\mu\lambda\psi$$

1. What do the spinors and the vector field represent?
2. Where is the non-Abelian nature of QCD 'hidden' in the Lagrangian density and how is it reflected on the gauge bosons of QCD?

(b) QCD has a number of unique features that the other theories of the Standard Model do not have. Asymptotic freedom and confinement are the most characteristic of them. What do they physically mean and what is their origin?

(c) The Standard Model has three quark generations but it does not tell us why we only have three. How many generations can theory accommodate for three colours before QCD stops being asymptotically free?

(d) Look carefully at figure 0.1

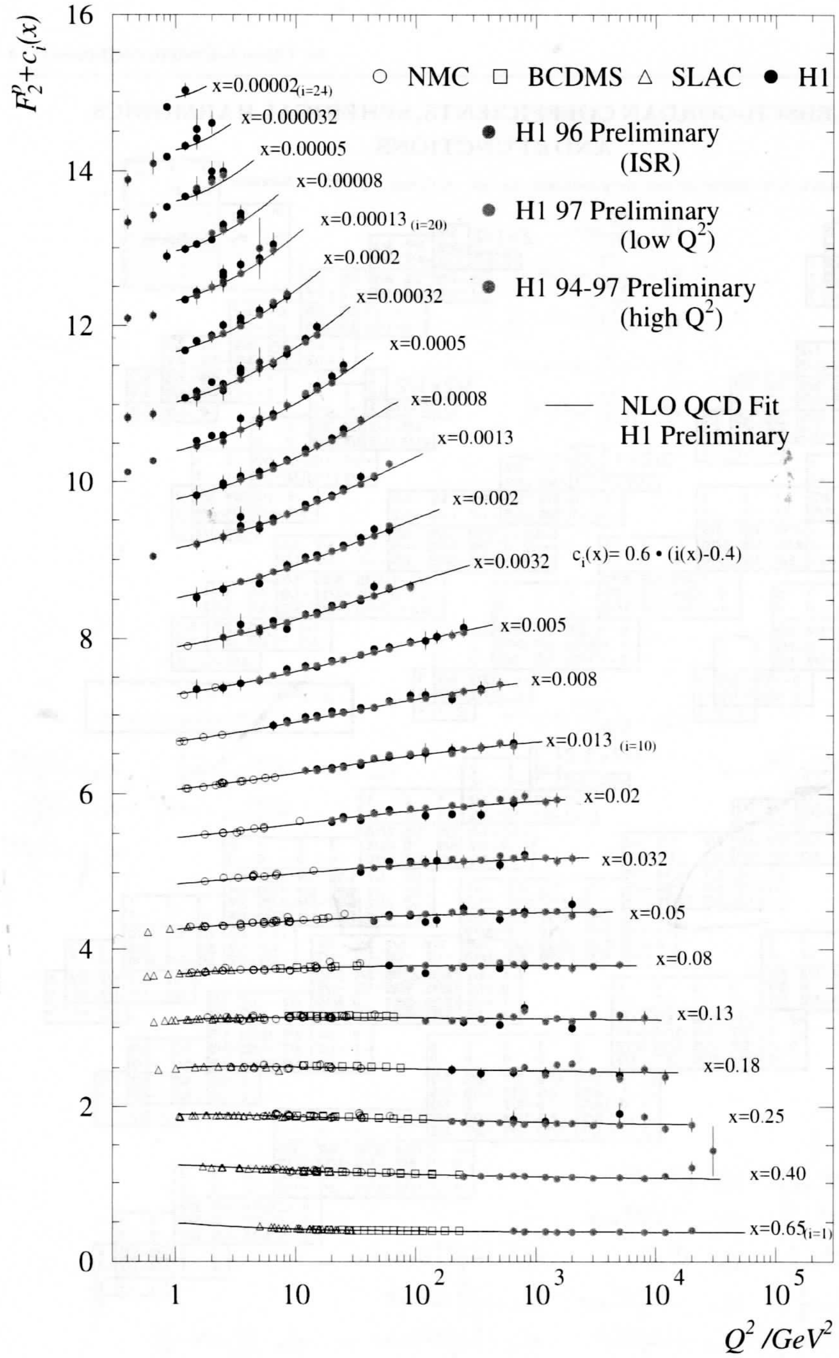
1. What is the physical meaning of the F_2 and Q^2 on the y- and x-axis, respectively?
2. What does this variable x represent i.e. what is its physical meaning?
3. What does the fact that F_2 does not depend on Q^2 for certain ranges of x values physically mean?
4. What does the breaking of this scaling for low values of x mean?

(e) Consider the process $e^+e^- \to q\bar{q}g$. The cross-section of such a process is given by

$$\frac{d^2\sigma}{dx_1 dx_2} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)},$$

where $\sigma_0 = \sigma(e^+e^- \to \text{hadrons}) = (4\pi\alpha^2/s)\sum e_i^2$, and $x_i = \frac{E_i}{\sqrt{s}/2}$ are the energy fractions of each outgoing parton.

1. Show that $\sum_{i=1}^{3} x_i = 2$.
2. Show that if one of the three partons is soft, the other two should come out in a back-to-back topology.
3. What happens to partons j and k if parton i has $x_i = 1$?

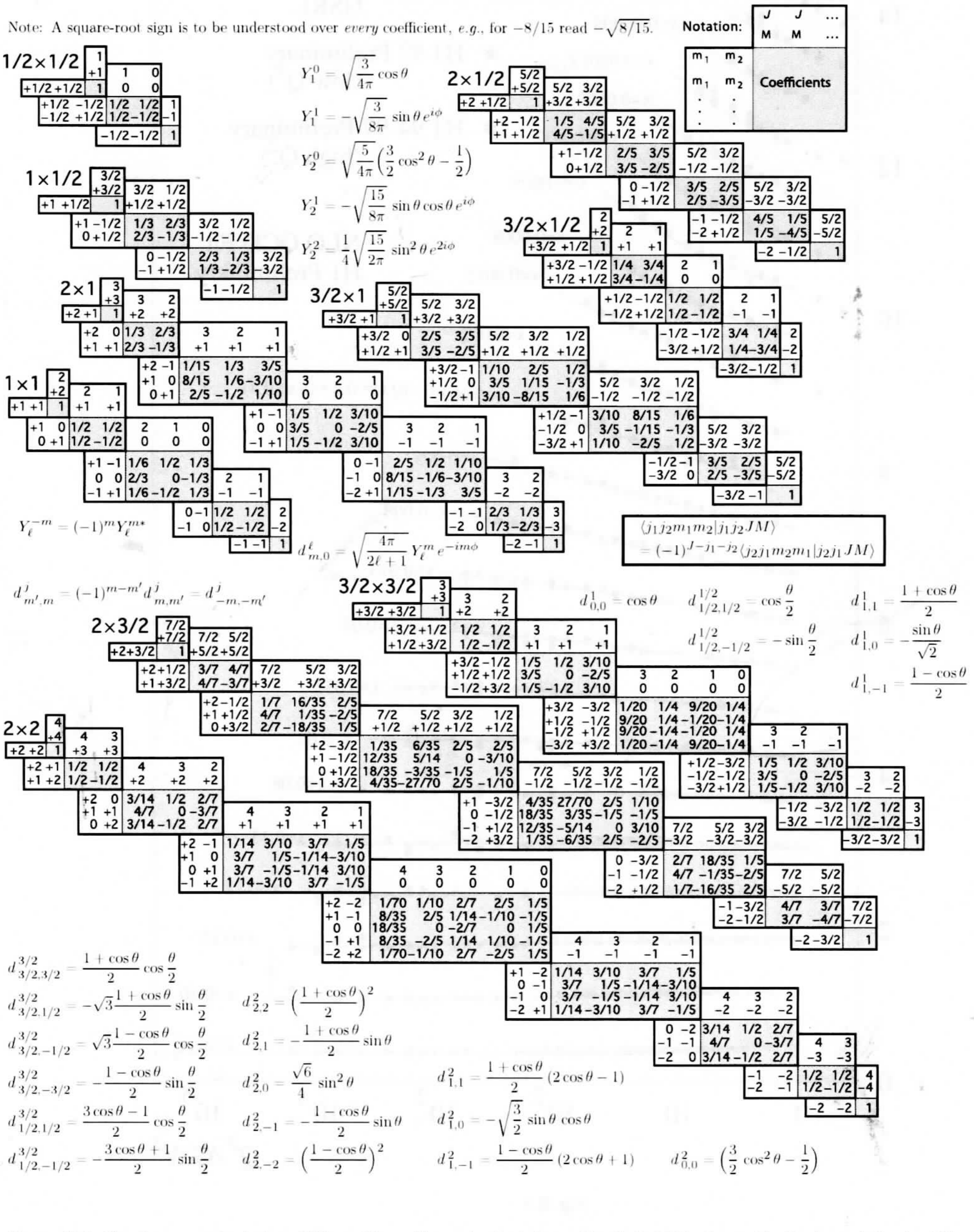


$F_2^p+c_i(x)$
NMC
BCDMS
SLAC
H1
H1 96 Preliminary (ISR)
H1 97 Preliminary (low Q²)
H1 94-97 Preliminary (high Q²)
NLO QCD Fit H1 Preliminary
$c_i(x)= 0.6 \cdot (i(x)-0.4)$
x=0.00002 (i=24)
x=0.000032
x=0.00005
x=0.00008
x=0.00013 (i=20)
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013 (i=10)
x=0.02
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.40
x=0.65 (i=1)
Q^2 /GeV²

Fig. 0.1

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.



$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

Fig. 0.2