Niels Tuning* and Panos Christakoglou†

Particle Physics II

Exam

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*h71@nikhef.nl - Nikhef, Science Park 105, 1098XG, Amsterdam, The Netherlands
†Panos.Christakoglou@nikhef.nl - Nikhef, Science Park 105, 1098XG, Amsterdam, The Netherlands

Every sub-question has the same absolute weight.

Exercise 1

Neutral Meson Mixing

Neutral heavy mesons can spontaneously oscillate into its own anti-particle and vice versa.

(a) Suppose one produces kaons in a pure flavour eigenstate, i.e. one starts with a pure $|K^0\rangle$ beam. What fraction of the kaons is CP-even ($|K^0_+\rangle$), and what fraction is CP-odd ($|K^0_-\rangle$), given

$$|K^0_+\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K}^0\rangle\right] \quad ; \quad |K^0_-\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K}^0\rangle\right].$$

(b) Assuming no CP-violation in mixing, these CP-eigenstates correspond to mass-eigenstates with a well-defined lifetime. At very large decay times (>1 ns), what fraction of our kaon beam is CP-odd?

(c) To observe CP violation in a given process, two (interfering) amplitudes are needed. To observe CP violation, what can you say about the relative phase of the two amplitudes? Cronin and Fitch observed CP violation in kaon mixing. Describe in words the two interfering amplitudes that are responsible for this CP violating effect.

(d) Draw the box diagram responsible for K^0 mixing. Which internal quark type gives the dominant contribution to the box diagram? Explain why.

Exercise 2

A CP measurement with the decays $B^0_s \to \phi\phi$.

(a) Argue that the final state $f = \phi\phi$ is a CP-eigenstate.

(b) Draw the Feynman diagrams for the decays $B^0_s \to \phi\phi$ and $\bar{B}^0_s \to \phi\phi$. Indicate the appropriate CKM-elements at the vertices.

(c) What is a penguin diagram? loop diagram using w and gluon

(d) Draw the diagram for the process where the B^0_s oscillates before it decays, $B^0_s \to \bar{B}^0_s \to \phi\phi$. Indicate the appropriate CKM-elements at the vertices.

(e) Finally, also draw the diagram of the CP-conjugate process $\bar{B}^0_s \to B^0_s \to \phi\phi$.

(f) What are the weak phases of each of the four amplitudes (in the Wolfenstein parameterization) ?

(g) We will now investigate the CP-asymmetry in this decay. An important quantity in the description of the CP-asymmetry of a decay to a final state f is: $\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$. The value of $\frac{q}{p}$ describes the mixing and can be assumed to be $|\frac{q}{p}| = 1$ in the B^0_s-system. What is the value of $\frac{\bar{A}_f}{A_f}$ for the process $B^0_s(\bar{B}^0_s) \to \phi\phi$?

(h) The expression for the time-dependent CP-asymmetry is given by, (with $f = \phi\phi$):

$$A_{CP}(t) = \frac{\Gamma_{B^0_s(t)\to f} - \Gamma_{\bar{B}^0_s(t)\to f}}{\Gamma_{B^0_s(t)\to f} + \Gamma_{\bar{B}^0_s(t)\to f}}$$

Write the expression for $A_{CP}(t)$ in terms of λ_f, t, Δm and $\Delta\Gamma$.

(i) Use the answer of (f) and give an expression for $\Im\lambda_f$.

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Exercise 3

Transformations, group theory and the colour quantum number

(a)Consider the operator $\hat{U} = e^{-i\alpha\hat{F}/\hbar}$, where α is a displacement in real space along the z axis and $\hat{F}$ the generator of the transformation.

1. What operation is described by U?
2. Identify the symmetry generator of the transformation if the Hamiltonian of the system is invariant under translation in space.
3. What is the conserved quantity?

(b)Confirm that the generators of the SU(2) group that describe the transformations in the spin and isospin space satisfy the SU(2) algebra $[\hat{X}^i, \hat{X}^j] = i\varepsilon_{ijk}\hat{X}^k$.

(c)Consider a particle of spin 3/2 and another one of spin 2 that form a system whose orbital angular momentum is 0 and total spin is 5/2. If the z-component of the composite system is -1/2, what values would we get for the measurement of S_z and what is the probability for each? Show that they add up to unity?

(d)The extension of SU(2) in the colour space leads to SU(3). For the SU(3) group, the infinitesimal transformation acting on a general three-component column vector has generators that are associated with the 8 Gell-Mann matrices. Associate each matrix to a gluon and explain the connection.

(e)There are nine gluon species which are described by the SU(3) color symmetry:

$$3 \otimes \bar{3} = 8 \oplus 1$$

Why do we only consider 8 of them as physical? Why is the singlet state excluded?

(f) Calculate the colour factors for the following state transition: $(BR + RB)/\sqrt{2} \to (BR + RB)/\sqrt{2}$

(g)Are the colour interactions between a quark and an antiquark attractive or repulsive and why?

For these you have to consider that the colour matrices are given by:

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \text{ for R, } \begin{pmatrix}0\\1\\0\end{pmatrix} \text{ for B, } \begin{pmatrix}0\\0\\1\end{pmatrix} \text{ for G}$$

The Gell-Mann matrices are:

$$\underbrace{\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}}_{\lambda_1} \underbrace{\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}}_{\lambda_2} \underbrace{\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}}_{\lambda_3} \underbrace{\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}}_{\lambda_4}$$

$$\underbrace{\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}}_{\lambda_5} \underbrace{\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}}_{\lambda_6} \underbrace{\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}}_{\lambda_7} \underbrace{\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}}_{\lambda_8}$$

Exercise 4

The QCD Lagrangian and the running coupling constant

(a)The QCD Lagrangian in the Standard Model is given by

$$\mathscr{L}_{\text{QCD}} = -\tfrac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ - m)\psi - (g_s\,\overline{\psi}\gamma^{\mu}\lambda\,\psi)\cdot A_{\mu}$$

Explain what each term represents.

(b)The QCD Lagrangian looks quite similar to the QED one, given by

$$\mathscr{L}_{\text{QED}} = -\tfrac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ - m)\psi - (g\,\overline{\psi}\gamma^{\mu}\psi)\cdot A_{\mu}$$

Identify and discuss the differences between the two Lagrangians for every single term.

(c)When one considers higher order corrections to the electron-muon scattering the propagator is modified accordingly to include one loop factors. After integration, the tensor $I_{\mu\nu}$ that describes this correction can be parameterised as

$$I_{\mu\nu} = -ig_{\mu\nu}q^2 I(q^2) + q_{\mu}q_{\nu}J(q^2)$$

,

Show that the second part of this parameterisation does not contribute to M_{if}.

Hint: replace $I_{\mu\nu}$ in the expression of M_{if} and use the Dirac equations of the relevant spinors:

$$(P\!\!\!/ - m)u = 0$$

$$\bar{u}(P\!\!\!/ - m) = 0$$

$$(P\!\!\!/ + m)v = 0$$

$$\bar{v}(P\!\!\!/ + m) = 0$$

(d)QCD has a number of unique features that the other theories of the Standard Model do not have. Asymptotic freedom and confinement are the most characteristic of them.

1. Explain what do they physically mean,
2. and where do they originate from.

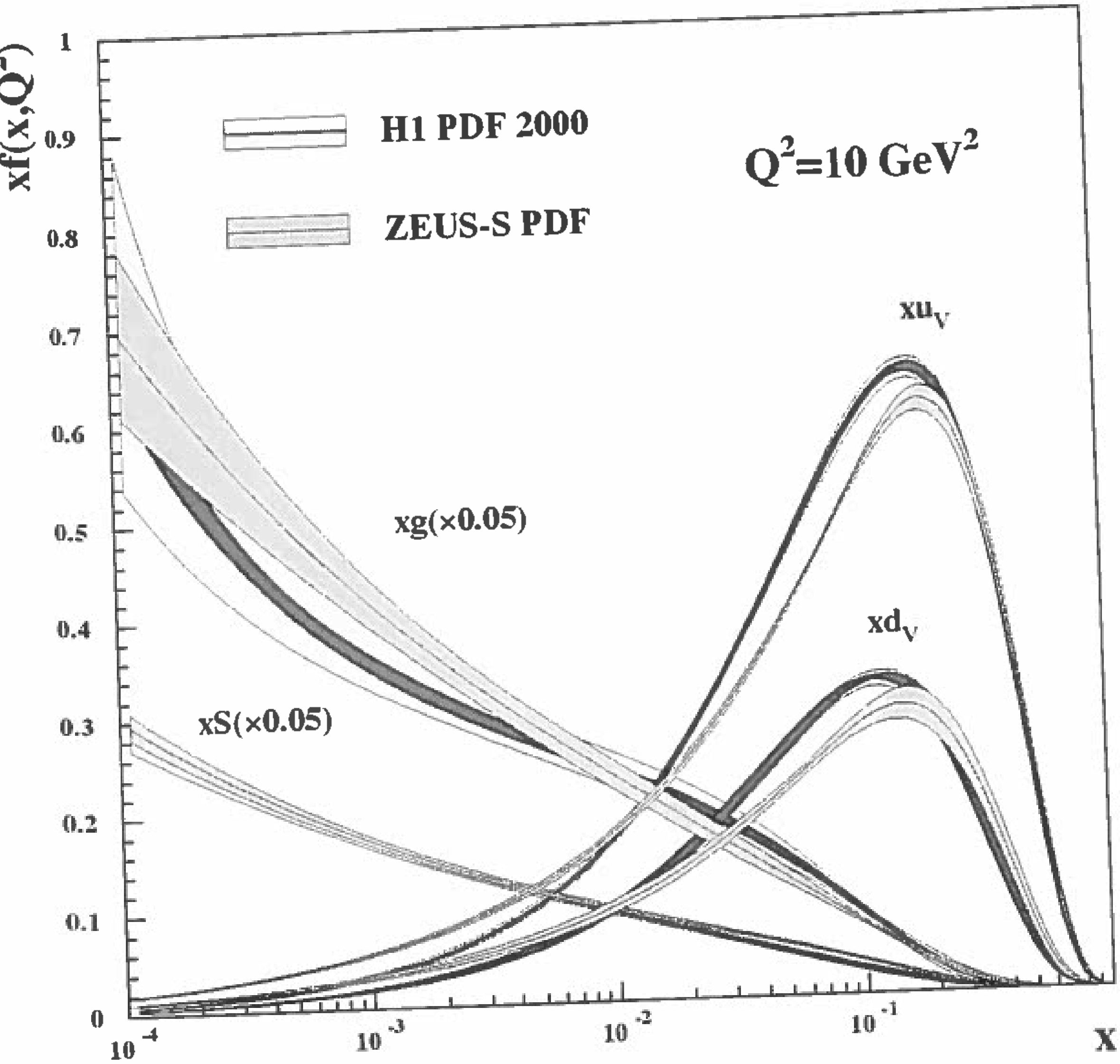
(e)For $N_c = 3$ determine an upper limit for n_f for which QCD is still asymptotically free.

(f) How can we probe experimentally the quark and gluon distribution functions?

(g)Look carefully at the plot below

1. What is plotted on the x- and what on the y-axis?
2. Write down the definition of the variable of the x-axis.
3. What does the variable on the x-axis represent i.e. what is its physical meaning?
4. What is the physical meaning of Q^2, written in the legend of the plot?
5. Find the value of the strong coupling strength for this value of Q^2 considering that the QCD scale, Λ_{QCD}, is around 200 MeV.
6. What are the different curves indicated as xg, xS, xu_v and xd_v?
7. What does the plot tell us?
8. If you would like to probe $g(x)$ at which x-values would you aim and why?

coupling varies large Q^2



xf(x,Q²)
H1 PDF 2000
ZEUS-S PDF
Q²=10 GeV²
xu_V
xd_V
xg(×0.05)
xS(×0.05)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10⁻⁴
10⁻³
10⁻²
10⁻¹
x

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J J ...	
	M M ...	
m_1 m_2		
m_1 m_2	Coefficients	
. .		

1/2×1/2

m_1	m_2	M	$J=1$	$J=0$
+1/2	+1/2	+1	1	
+1/2	−1/2	0	1/2	1/2
−1/2	+1/2	0	1/2	−1/2
−1/2	−1/2	−1	1	

1×1/2

m_1	m_2	M	$J=3/2$	$J=1/2$
+1	+1/2	+3/2	1	
+1	−1/2	+1/2	1/3	2/3
0	+1/2	+1/2	2/3	−1/3
0	−1/2	−1/2	2/3	1/3
−1	+1/2	−1/2	1/3	−2/3
−1	−1/2	−3/2	1	

2×1/2

m_1	m_2	M	$J=5/2$	$J=3/2$
+2	+1/2	+5/2	1	
+2	−1/2	+3/2	1/5	4/5
+1	+1/2	+3/2	4/5	−1/5
+1	−1/2	+1/2	2/5	3/5
0	+1/2	+1/2	3/5	−2/5
0	−1/2	−1/2	3/5	2/5
−1	+1/2	−1/2	2/5	−3/5
−1	−1/2	−3/2	4/5	1/5
−2	+1/2	−3/2	1/5	−4/5
−2	−1/2	−5/2	1	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

3/2×1/2

m_1	m_2	M	$J=2$	$J=1$
+3/2	+1/2	+2	1	
+3/2	−1/2	+1	1/4	3/4
+1/2	+1/2	+1	3/4	−1/4
+1/2	−1/2	0	1/2	1/2
−1/2	+1/2	0	1/2	−1/2
−1/2	−1/2	−1	3/4	1/4
−3/2	+1/2	−1	1/4	−3/4
−3/2	−1/2	−2	1	

2×1

m_1	m_2	M	$J=3$	$J=2$	$J=1$
+2	+1	+3	1		
+2	0	+2	1/3	2/3	
+1	+1	+2	2/3	−1/3	
+2	−1	+1	1/15	1/3	3/5
+1	0	+1	8/15	1/6	−3/10
0	+1	+1	2/5	−1/2	1/10
+1	−1	0	1/5	1/2	3/10
0	0	0	3/5	0	−2/5
−1	+1	0	1/5	−1/2	3/10
0	−1	−1	2/5	1/2	1/10
−1	0	−1	8/15	−1/6	−3/10
−2	+1	−1	1/15	−1/3	3/5
−1	−1	−2	2/3	1/3	
−2	0	−2	1/3	−2/3	
−2	−1	−3	1		

1×1

m_1	m_2	M	$J=2$	$J=1$	$J=0$
+1	+1	+2	1		
+1	0	+1	1/2	1/2	
0	+1	+1	1/2	−1/2	
+1	−1	0	1/6	1/2	1/3
0	0	0	2/3	0	−1/3
−1	+1	0	1/6	−1/2	1/3
0	−1	−1	1/2	1/2	
−1	0	−1	1/2	−1/2	
−1	−1	−2	1		

3/2×1

m_1	m_2	M	$J=5/2$	$J=3/2$	$J=1/2$
+3/2	+1	+5/2	1		
+3/2	0	+3/2	2/5	3/5	
+1/2	+1	+3/2	3/5	−2/5	
+3/2	−1	+1/2	1/10	2/5	1/2
+1/2	0	+1/2	3/5	1/15	−1/3
−1/2	+1	+1/2	3/10	−8/15	1/6
+1/2	−1	−1/2	3/10	8/15	1/6
−1/2	0	−1/2	3/5	−1/15	−1/3
−3/2	+1	−1/2	1/10	−2/5	1/2
−1/2	−1	−3/2	3/5	2/5	
−3/2	0	−3/2	2/5	−3/5	
−3/2	−1	−5/2	1		

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

m_1	m_2	M	$J=7/2$	$J=5/2$	$J=3/2$	$J=1/2$
+2	+3/2	+7/2	1			
+2	+1/2	+5/2	3/7	4/7		
+1	+3/2	+5/2	4/7	−3/7		
+2	−1/2	+3/2	1/7	16/35	2/5	
+1	+1/2	+3/2	4/7	1/35	−2/5	
0	+3/2	+3/2	2/7	−18/35	1/5	
+2	−3/2	+1/2	1/35	6/35	2/5	2/5
+1	−1/2	+1/2	12/35	5/14	0	−3/10
0	+1/2	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	+1/2	4/35	−27/70	2/5	−1/10
+1	−3/2	−1/2	4/35	27/70	2/5	1/10
0	−1/2	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	−1/2	12/35	−5/14	0	3/10
−2	+3/2	−1/2	1/35	−6/35	2/5	−2/5
0	−3/2	−3/2	2/7	18/35	1/5	
−1	−1/2	−3/2	4/7	−1/35	−2/5	
−2	+1/2	−3/2	1/7	−16/35	2/5	
−1	−3/2	−5/2	4/7	3/7		
−2	−1/2	−5/2	3/7	−4/7		
−2	−3/2	−7/2	1			

3/2×3/2

m_1	m_2	M	$J=3$	$J=2$	$J=1$	$J=0$
+3/2	+3/2	+3	1			
+3/2	+1/2	+2	1/2	1/2		
+1/2	+3/2	+2	1/2	−1/2		
+3/2	−1/2	+1	1/5	1/2	3/10	
+1/2	+1/2	+1	3/5	0	−2/5	
−1/2	+3/2	+1	1/5	−1/2	3/10	
+3/2	−3/2	0	1/20	1/4	9/20	1/4
+1/2	−1/2	0	9/20	1/4	−1/20	−1/4
−1/2	+1/2	0	9/20	−1/4	−1/20	1/4
−3/2	+3/2	0	1/20	−1/4	9/20	−1/4
+1/2	−3/2	−1	1/5	1/2	3/10	
−1/2	−1/2	−1	3/5	0	−2/5	
−3/2	+1/2	−1	1/5	−1/2	3/10	
−1/2	−3/2	−2	1/2	1/2		
−3/2	−1/2	−2	1/2	−1/2		
−3/2	−3/2	−3	1			

2×2

m_1	m_2	M	$J=4$	$J=3$	$J=2$	$J=1$	$J=0$
+2	+2	+4	1				
+2	+1	+3	1/2	1/2			
+1	+2	+3	1/2	−1/2			
+2	0	+2	3/14	1/2	2/7		
+1	+1	+2	4/7	0	−3/7		
0	+2	+2	3/14	−1/2	2/7		
+2	−1	+1	1/14	3/10	3/7	1/5	
+1	0	+1	3/7	1/5	−1/14	−3/10	
0	+1	+1	3/7	−1/5	−1/14	3/10	
−1	+2	+1	1/14	−3/10	3/7	−1/5	
+2	−2	0	1/70	1/10	2/7	2/5	1/5
+1	−1	0	8/35	2/5	1/14	−1/10	−1/5
0	0	0	18/35	0	−2/7	0	1/5
−1	+1	0	8/35	−2/5	1/14	1/10	−1/5
−2	+2	0	1/70	−1/10	2/7	−2/5	1/5
+1	−2	−1	1/14	3/10	3/7	1/5	
0	−1	−1	3/7	1/5	−1/14	−3/10	
−1	0	−1	3/7	−1/5	−1/14	3/10	
−2	+1	−1	1/14	−3/10	3/7	−1/5	
0	−2	−2	3/14	1/2	2/7		
−1	−1	−2	4/7	0	−3/7		
−2	0	−2	3/14	−1/2	2/7		
−1	−2	−3	1/2	1/2			
−2	−1	−3	1/2	−1/2			
−2	−2	−4	1				

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).