MCM 2018-19, End-term exam

Monday, 28th of January 2019. Hand in before 12:00

Please write clearly and enclose your final answers in a [box]. This test has 4 problems.

1. **Binding energy in doped semiconductors**
   The binding energy of electrons to the proton in a Hydrogen atom are given by the Rydberg series $E_H = -R_y/n^2$ with n the principle orbital number and $R_y = 13.6$ eV the Rydberg energy. The mean separation between electron and nucleus in each orbital is given by the Bohr radius $a_B = 0.51 \times 10^{-10}$ m. From quantum mechanics we know $R_y = \frac{me^4}{8\epsilon_0^2 h^2}$ and $a_B = \frac{\epsilon_0 h^2}{\pi m e^2}$, with m and e the mass of the electron and the elementary charge, h the Planck constant ϵ_0 the vacuum permeability.

   (a) What is the binding energy of electrons to Phosphorus donor atoms in Silicon, given dielectric constant of $\epsilon = 11.7$ and effective electron mass of $m^* = 0.19\,m$ in the bottom of the conduction band? Justify your answer.

   (b) What is the orbiting radius of the bound electrons in (a)?

2. **Nanoscience: single electron transistor**
   A single electron transistor, in its simplest form, consists of a very small metallic island between two metallic electrodes, isolated by a very thin layer of insulator. The electrostatic potential of the island can be controlled by a gate electrode at potential V_g. Electrons can tunnel through these barriers into and out of the island.

   (a) Write down the electrostatic energy (neglect the quantum effects) of the island as a function of the number of electrons on the island, its capacity C, and the gate potential. What is the required energy for adding one extra electron to the island if there are already N electrons present?

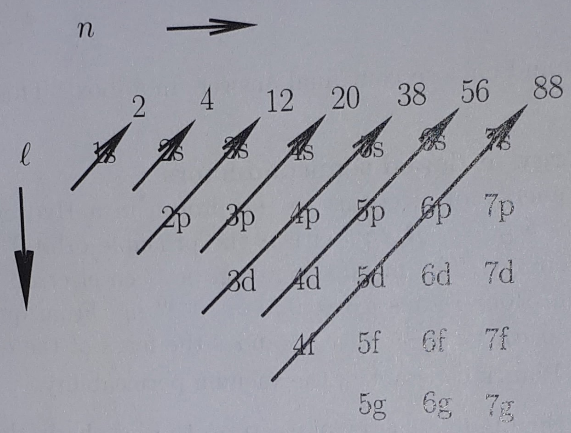
   (b) Due to the Coluomb interaction, the electronic energy levels of the island are quantised. Give an estimate for the ground state energy and the energy of the first excited state for the cases of $N = 1$ and $N = 2$ while $V_g = 0$.

   (c) Under what conditions for the chemical potentials of the two electrodes μ_R and μ_L, the gate potential, and the energy levels of the island, could there be a current passing through the island?

   (d) How can this geometry allow for only a single electron on the island at each moment? Explain qualitatively why this effect is most commonly observed at low temperatures.

3. **Magnetism in Dysprosium (Dy^{3+})**
In this exercise we are considering the magnetism of Dy^{3+}, which is one of the strongest magnetic element for the lanthanides with $\mu_{\text{eff}} = 10.5\ \mu_B$.

(a) Use the Aufbau principle to determine the electron configuration for this ion, which has a nuclear charge of Z=66. *Hint: Use the diagram below for the ordering of the orbitals, AND use the fact that despite this diagram the 4f-orbital is filled up before the 6s-orbital.*



To determine the ground state of the ion, Hund's rules are used. The Hund's rule state that

i. For given L the highest possible spin angular momentum S has the lowest energy.

ii. For given S the highest possible orbital angular momentum L has the lowest energy.

iii. The total angular momentum J is given for a less than half filled shell by $J = |L - S|$, whereas for a more than half filled shell $J = L + S$.

(b) Determine S, L and J for the ground state of the ion using these rules. *Hint, if you do not know the values of S, L and J, use in the following the* **wrong** *values for S = 3, L = 3 and J=7/2.*

The effective magnetic moment is given by $\mu_{\text{eff}} = g_j\sqrt{J(J+1)}\mu_B$, with the gyromagnetic ratio g_j given by

$$g_j = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}.$$

(c) Determine the effective magnetic moment of Dy^{3+}.

(d) What is the effective magnetic moment, if it is assumed that $L = 0$?

(e) Compare both results with the experimental value and discuss the result.

(f) Why do you think that Dy^{3+} is one of the strongest magnetic elements of the lanthanides? *Hint, you only have to provide qualitative arguments.*

(g) Why does the orbital angular momentum in Dy^{3+} contribute to the effective magnetic moment, whereas in Fe^{3+} it does not?

(c) the group velocity for the dispersion relation in (a) vanishes at $k = \pm\frac{\pi}{a}$, and

(d) ω is periodic in k with period $\frac{2\pi}{a}$.

(e) Also explain why you would expect (b), (c), and (d) to remain valid if forces between neighbours of even higher order are included.