MCM 2018-19, Mid-term exam

Wednesday, 12th of December. Hand in before 10:50

Thou shall not use small numbers to represent the whole. (Hasty generalization)

Please write clearly and enclose your final answers in a [box]. This test has 3 problems and in total 12 sub-items.

1. **Nearly-free electrons in a one-dimensional lattice**
   Consider electrons in a one-dimensional lattice of lattice constant a

   (a) Plot the energy dispersion relation of the free electrons in the first and the second Brillouin zones.

   (b) Assume that the interaction potential of the atoms with the electrons can be approximated with $U(x) = U_0(1 - \cos 2\pi x/a)$. Write down the central equation for the lowest energy eigenstate coefficients at the boundary to the 1st Brillouin zone

   (c) Calculate the band gap for the region mentioned in (b)

   (d) Similarly, calculate the band gap at $k = 0$ between the second and the third band, given the potential in (b).

2. **Semiconductor quantum well**
   A quantum well is formed from a layer of GaAs of thickness L nanometers, surrounded by thicker layers of $Ga_{1-x}Al_xAs$. You may assume that the band gap of the surrounding layers is substantially larger than that of GaAs. The electron effective mass in GaAs is $0.068\,m_e$ whereas the hole effective mass is $0.45\,m_e$, with m_e the mass of electron.

   (a) Sketch the shape of the potential for the electrons and holes.

   (b) What is the effect of the electron confinement in the potential well on the minimum energy of the conduction band. (Hint: expand the parabolic dispersion of electrons in terms the 3 momentum components and consider the effect of the potential well on each of the components).

   (c) What approximate value of L is required if the band gap of the quantum well is to be 0.1 eV larger than that of GaAs bulk material?

3. **Vibrations in a harmonic chain with next-nearest-neighbour coupling**
   Consider the lattice vibrations of a chain of identical masses m, in which each mass is connected to its first and second nearest neighbours by springs of spring constant K_1 and K_2 respectively, and the equilibrium spacing between neighboring masses is a. Show that

   (a) the lattice vibrations follow the dispersion relation

   $$m\omega^2 = 2K_1[1 - \cos(ka)] + 2K_2[1 - \cos(2ka)]$$

   (b) the dispersion relation in (a) reduces to that for sound waves in the long wavelength limit. What is the propagation velocity in this limit?

The magnetization M is given by

$$M = N\frac{\sum_{M_J=-J}^{+J}(-g_J\mu_B M_J)f(M_J)}{\sum_{M_J=-J}^{+J} f(M_J)},$$

with N the atomic density and the Boltzmann factor $f(M_J) = \exp(-g_j\mu_B M_J B/k_B T)$.

(h) Explain in physical terms the Boltzmann factor.

(i) Show that for large temperature that the susceptibility $\chi \equiv \mu_0 M/B$ is proportional to $1/T$.

(j) Under what condition is the result in the previous item valid?

4. **Different kinds of magnetism**

(a) Why is Fe^{3+} above the Curie temperature a paramagnet? Provide a physical reasoning.

(b) Why does Fe^{3+} become a ferromagnet below the Curie temperature? Provide a physical reasoning.

A ferromagnet below the Curie temperature can be described by the Heisenberg Hamiltonian:

$$\mathcal{H} = -J\sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j,$$

where the sum runs over only the nearest neighbors.

(c) Why do we neglect in the sum all the other atoms apart from the neareast neighbors?

(d) What is the microscopic nature of the interaction term J?

Spin waves can be excited in ferromagnets.

(e) Can we also excite spin waves above the Curie temperature? Explain your answer.

(f) Can spin waves also be induced in antiferromagnets? Explain your answer.

(g) How can spin waves be used for logic gates?