info AN prio 1

1 2nd Exam Algorithms and Networks 2008

You have two hours for the exam. You may give your answers in Dutch or in English. Write clearly. You may consult four sides of A4 with notes.

Results used in the course or exercises may be used without further proof, unless explicitly asked.

Some parts are harder than others: use your time well, and make sure you first finish the easier parts!

Good luck!

1. Force directed drawing (1.25 point) Explain in your own words how the *electrical forces and springs* method for graph drawing works.

2. Stanley-Gale algorithm (1.25 point) Consider the Stanley-Gale algorithm for the STABLE MARRIAGE problem. Suppose the algorithm runs on a set of n men and n women.

- Is the following proposition correct?

  Let M^* be a man. At most $n-1$ times, it can happen to M^* that he loses his current assignment, i.e., the event that M^* is assignment to a woman, and then M^* is no longer assigned to that woman, happens at most $n-1$ times.

- Prove that your answer is correct.

3. The number of independent sets in a binary tree ($3 = 0.5 + 0.5 + 0.75 + 0.25 + 1$ points) In this assignment, we design an algorithm to count the *number* of independent sets in a binary tree.

Let T be a binary tree with root r. Write $A(T)$ is the number of independent sets S in T with $r \in S$, and write $B(T)$ is the number of independent sets S in T with $r \notin S$.

1. Suppose T_1 is a binary tree with root r_1, T_2 is a binary tree with root r_2. Let T be the tree, formed by taking the disjoint union of T_1 and T_2, then adding a new vertex r and edges $\{r, r_1\}$ and $\{r, r_2\}$. r is the root of T. See Figure 1 for an example. Argue that the following formula holds:

$$B(T) = (A(T_1) + B(T_1)) \cdot (A(T_2) + B(T_2))$$

2. Give a formula that expresses $A(T)$ in terms of $A(T_1)$, $A(T_2)$, $B(T_1)$, and/or $B(T_2)$. (You do not need to argue correctness.)

3. Suppose T_1 is a binary tree with root r_1, and T^* is formed by adding a new vertex r and edge $\{r, r_1\}$ to T_1. r is the root of T^*. See Figure 2 for an example. Give formulas that express $A(T^*)$ and $B(T^*)$ in terms of $A(T_1)$ and/or $B(T_1)$.

5. Cluster deletion (2 points) Consider the following parameterized problem:

CLUSTER DELETION
Given: Undirected graph $G = (V, E)$, integer K.
Parameter: K
Question: Can we delete at most K edges such that each connected component is a clique, i.e., is there a set $F \subseteq E$ with $|F| \leq K$, and each connected component of $G' = (V, E - F)$ is a clique?

Show that this problem belongs to the class FPT.