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INFOB3TC – Solutions for Exam 2

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Please keep in mind that there are often many possible solutions and that these example solutions may contain mistakes.

Questions

Regular expressions, languages and pumping lemmas

1 (10 points)**.** Consider the DFA (X, Q, d, S, F) where $X = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, $Q = \{q_1, q_2\}$, d is defined by:

$$
\begin{aligned}
&d\ q_1\ \mathtt{a} = q_1\\
&d\ q_1\ \mathtt{b} = q_2\\
&d\ q_2\ \mathtt{a} = q_1\\
&d\ q_2\ \mathtt{c} = q_2
\end{aligned}
$$

$S = q_1$, and $F = \{q_2\}$. Give the regular expression denoting the language accepted by this automaton. •

Solution 1.

$\mathtt{a}^*\mathtt{b}(\mathtt{a}^+\mathtt{b} + \mathtt{c})^*$

Marking

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For the following three tasks: consider the following three languages:

$$
\begin{aligned}
L_1 &= \{a^n b^m c d^m e^n \mid n, m \geqslant 0\}\\
L_2 &= \{(ab)^n c d^m \mid n, m \geqslant 0\}\\
L_3 &= \{a^n b (cd)^n e^n \mid n \geqslant 0\}
\end{aligned}
$$

2 (5 points). One of the languages is regular, one context-free and not regular and one not context-free. Which are the regular and the non-regular context-free languages? •

Solution 2.
L_2 is regular, L_1 is context-free, L_3 is neither. ◦

3 (5 points). Give a regular grammar for the regular language, and a context-free for the context-free language. •

Solution 3.

Here is a regular grammar for L_2:

$S \to abS$
$S \to c$
$S \to cB$
$B \to d$
$B \to dB$

Here is a context-free grammar for L_1:

$S \to aSe$
$S \to T$
$T \to bTd$
$T \to c$

◦

4 (10 points). Prove that the grammar that is context-free but not regular is indeed not regular by using the pumping lemma for regular languages . •

Solution 4.

The language L_1 is not regular. To prove it, we assume it is regular and find a contradiction using the pumping lemma.

For any n,
let $x = \varepsilon$, $y = a^n$, $z = b^m c d^m e^n$.
Then, $xyz = a^n b^m c d^m e^n \in L_1$ and $|y| \geqslant n$.

From the pumping lemma, we know there must be a loop in y, i.e. $y = uvw$ with $q = |v| > 0$ such that $xuv^i wz \in L_1$ for all $i \in \mathbb{N}$.

Let $i = 2$. We expect $xuv^2wz \in L_1$. If $u = a^s$, $v = a^q$, $w = a^t$, then we get $a^{s+2\,q+t} b^m c d^m e^n = a^{n+q} b^m c d^m e^n \in L$. But this word is not in L, since $q > 0$. Therefore, L_1 is not regular.

Marking

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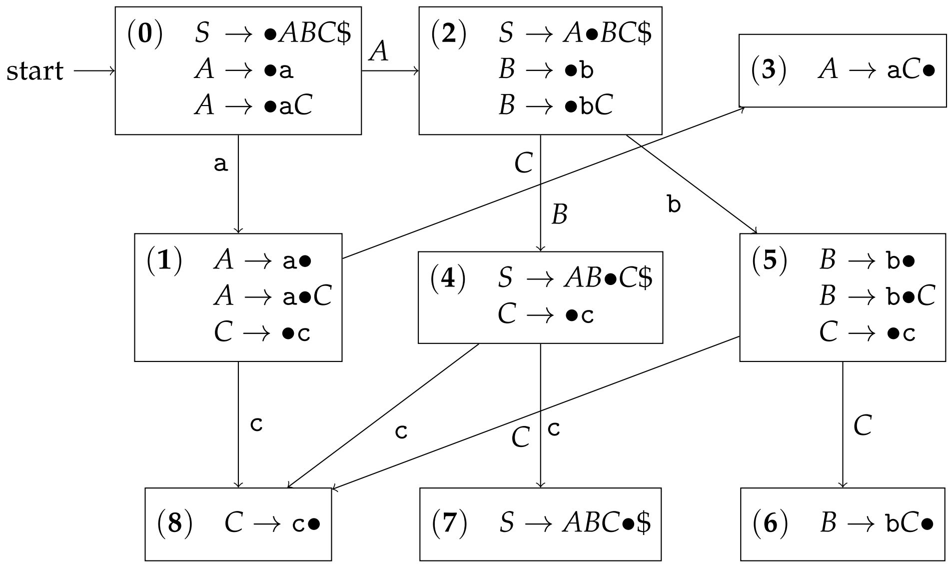
LR parsing

Consider the following grammar:

$$S \to ABC\$$$
$$A \to \texttt{a}$$
$$A \to \texttt{a}C$$
$$B \to \texttt{b}$$
$$B \to \texttt{b}C$$
$$C \to \texttt{c}$$

5 (10 points)**.** This grammar is not LR(0). Construct the LR(0) automaton for this grammar, and show which conflicts appear where. •

Solution 5.



States (**1**) and (**5**) have a shift/reduce conflict.

Marking

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6 (10 points)**.** Is this grammar SLR(1)? If so, construct the SLR-table. If not, explain where you cannot make a choice in a shift/reduce conflict or a reduce/reduce conflict. •

Solution 6. This grammar is not SLR(1). The follow symbol of A is `b`, so the conflict in state (**1**) can be resolved: shift if you see a `c` in the input, reduce if you see a `b`. The follow symbol of B is `c`, so the conflict in state (**5**) cannot be resolved.

Marking

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7 (10 points)**.** Play through the LR parsing process for the sentence "acbcc\$". If there is a choice somewhere, make this explicit. Show in each step at which state in your LR(0) automaton you are. •

Solution 7.

stack	input	remark
$(\mathbf{0})$	acbcc\$	shift
$(\mathbf{0})\mathtt{a}(\mathbf{1})$	cbcc\$	shift (choice)
$(\mathbf{0})\mathtt{a}(\mathbf{1})\ \mathtt{c}(\mathbf{8})$	bcc\$	reduce by $C \to \mathtt{c}$
$(\mathbf{0})\mathtt{a}(\mathbf{1})\ C(\mathbf{3})$	bcc\$	reduce by $A \to \mathtt{a}C$
$(\mathbf{0})A(\mathbf{2})$	bcc\$	shift
$(\mathbf{0})A(\mathbf{2})\ \mathtt{b}(\mathbf{5})$	cc\$	shift (choice)
$(\mathbf{0})A(\mathbf{2})\ \mathtt{b}(\mathbf{5})\ \mathtt{c}(\mathbf{8})$	c\$	reduce by $C \to \mathtt{c}$
$(\mathbf{0})A(\mathbf{2})\ \mathtt{b}(\mathbf{5})\ C(\mathbf{6})$	c\$	reduce by $B \to \mathtt{b}C$
$(\mathbf{0})A(\mathbf{2})\ B(\mathbf{4})$	c\$	shift
$(\mathbf{0})A(\mathbf{2})\ B(\mathbf{4})\ \mathtt{c}(\mathbf{8})$	\$	reduce by $C \to \mathtt{c}$
$(\mathbf{0})A(\mathbf{2})\ B(\mathbf{4})\ C(\mathbf{7})$	\$	shift
$(\mathbf{0})A(\mathbf{2})\ B(\mathbf{4})\ C(\mathbf{7})\$$		reduce by $S \to ABC\$$
S		accept

Marking

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LL parsing

In these exercises we will look at the grammar

$$S \to AB$$
$$A \to \mathtt{a}A\mathtt{a} \mid \varepsilon$$
$$B \to \mathtt{b}B\mathtt{b} \mid \varepsilon$$

8 (15 points)**.** Complete the table below by computing the values in the columns for the appropriate rows. Use *True* and *False* for property values and set notation for everything else.

NT	Production	*empty*	*emptyRhs*	*first*	*firstRhs*	*follow*	*lookAhead*
S							
	$A \to AB$						
A							
	$A \to \mathtt{a}A\mathtt{a}$						
	$A \to \varepsilon$						
B							
	$B \to \mathtt{b}B\mathtt{b}$						
	$B \to \varepsilon$						

●

Solution 8.

NT	Production	*empty*	*emptyRhs*	*first*	*firstRhs*	*follow*	*lookAhead*
S		True		$\{\mathtt{a},\mathtt{b}\}$		$\{\}$	
	$S \to AB$		True		$\{\mathtt{a},\mathtt{b}\}$		$\{\mathtt{a},\mathtt{b}\}$
A		True		$\{\mathtt{a}\}$		$\{\mathtt{a},\mathtt{b}\}$	
	$A \to \mathtt{a}A\mathtt{a}$		False		$\{\mathtt{a}\}$		$\{\mathtt{a}\}$
	$A \to \varepsilon$		True		$\{\}$		$\{\mathtt{a},\mathtt{b}\}$
B		True		$\{\mathtt{b}\}$		$\{\mathtt{b}\}$	
	$B \to \mathtt{b}B\mathtt{b}$		False		$\{\mathtt{b}\}$		$\{\mathtt{b}\}$
	$B \to \varepsilon$		True		$\{\}$		$\{\mathtt{b}\}$

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9 (10 points)**.** Is the above grammar LL(1)? Explain how you arrived at your answer. If the grammar is not LL(1), give a grammar that generates the same language and is LL(1). ●

Solution 9.

The above grammar is not LL(1) because the *lookAhead* sets of the A and B productions have a non-empty intersection. The following grammar generates the same language and is LL(1).

$$S \to AB$$
$$A \to \mathtt{aa}A \mid \varepsilon$$
$$B \to \mathtt{bb}B \mid \varepsilon$$

Since *Follow* $(A) = \{\mathtt{b}\}$, and *Follow* $(B) = \{\,\}$, the intersections of lookahead sets of the productions for A and B, respectively, are empty.

Marking

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10 (5 points)**.** Show the steps that a parser for the above LL(1) grammar goes through to recognize the following input sequence:

`aabb`

For each step (one per line), show the stack, the remaining input, and the action (followed by the relevant symbol or production) performed. If you reach a step in which you cannot proceed, note the action as "error." •

Solution 10.

stack	**input**	**action**
S	aabb	initial state
AB	aabb	expand S
aaAB	aabb	expand A
AB	bb	match (2×)
B	bb	expand A
bb$B$	bb	expand B
ε	ε	match (2×)

Marking

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