

Final Exam Geometric Algorithms, March 14, 2007, 9–12

Read every question carefully, make sure you understand it, and be sure to answer the question. Read the question again after answering it, as a check. Make the easier questions first, and then the harder ones. You may not use any algorithmic result from the book, unless it is explicitly stated that this is allowed in the question. Answer questions in sufficient but not too much detail. You may not use the book during the exam.

Be sure to put your name on every piece of paper you hand in. Also write down your “collegekaartnummer”. Good luck!

1. (1.5 points)

(a) Let P be a set of points that all lie on the upper convex hull. What can you say about the lines dual to these points?

(b) Let S be a set of disjoint line segments that lie on the same (non-vertical) line ℓ . Let ℓ' be a line not parallel to ℓ and not vertical, and let ℓ' lie above all line segments of S . What is the dual situation for S , ℓ , and ℓ' ?

2. (1.5 points)

Here is a paranoid algorithm to compute the maximum of a set A of n real numbers:

Algorithm PARANOIDMAXIMUM(A)

1. **if** $\text{card}(A) = 1$ // the cardinality of A
2. **then return** the unique element $x \in A$
3. **else** Pick a random element x from A .
4. $x' \rightarrow \text{PARANOIDMAXIMUM}(A \setminus \{x\})$
5. **if** $x \leq x'$
6. **then return** x'
7. **else** Now we suspect that x is the maximum, but
 to be absolutely sure, we compare x with all
 $\text{card}(A) - 1$ other elements of A .
8. **return** x

Analyse the worst-case and the expected running time of this algorithm.

3. (1 point)

The search structure that is made during the randomized incremental construction algorithm for planar point location is a directed acyclic graph. It is not a tree, because there may be undirected cycles. In what situation(s) in the construction algorithm are undirected cycles made?

4. (1 point)

Suppose the Delaunay triangulation \mathcal{T} of a set P of n points in the plane is given. We want to compute for each point of P which other point of P is closest to it (so the output consists of n pairs of points, where the first points of all pairs are different). Sketch an efficient algorithm that, given \mathcal{T} , solves this problem, and analyze its running time.

5. (2 points)

Let T be a set of n disjoint triangles and let C be a set of n disjoint unit circles in the plane. Give a plane sweep algorithm that reports in $O(n \log n)$ time *the number of* triangles in T that (entirely) contain at least one circle from C . Define the status, describe the status structure, define the events that your algorithm will have, and how they are treated.

Analyze the time taken for handling each event, and use this to justify that the overall running time of the algorithm is $O(n \log n)$.

6. (2 points)

(a) Describe a 1D interval tree storing n intervals. What is the storage requirement? Describe a query with a point for all intervals containing that point. What is the query time?

(b) Describe a 1D segment tree storing n intervals. What is the storage requirement? Describe a query with a point for all intervals containing that point. What is the query time?

7. (1 point)

A rectilinear polygon is a simple polygon of which all edges are horizontal or vertical. Give an example to show that $\lfloor n/4 \rfloor$ cameras may be necessary to guard a rectilinear polygon with n vertices.